We give the existence result and the vanishing order of the solution in 0 for the following equation: \(-\Delta u(x) + (\mu/|x|^2)u(x) = \lambda u(x) + u^{2^*-1}(x)\), where \(x \in B_1\), \(\mu > 0\), and the potential \(\mu/|x|^2 - \lambda\) is positive in \(B_1\).

1. Introduction

In this paper, we consider the following problem:

\[-\Delta u(x) + \frac{\mu}{|x|^2}u(x) = \lambda u(x) + u^{2^*-1}(x), \quad x \in B_1,\]

\[u(x) \geq 0, \quad x \in B_1,\]

\[u(x) = 0, \quad x \in \partial B_1,\]

where \(B_1 = \{x \in \mathbb{R}^N \mid |x| < 1\}\) is the unit ball in \(\mathbb{R}^N (N \geq 3)\), \(\lambda, \mu > 0, 2^* := 2N/(N - 2)\). When \(\mu < 0\), this problem has been considered by many authors recently (cf. [5, 6, 7, 8]). But when \(\mu > 0\), this problem has not been considered as far as we know. In fact, the existence of nontrivial solution for (1.1) when \(\mu > 0\) is an open problem which was imposed in [7]. In this paper, we get the following results.

**Theorem 1.1.** If \(N = 3\) and \(3/4 < \lambda \leq \mu\) or if \(N \geq 4\) and \(0 < \lambda \leq \mu\), then for (1.1) there exists a nontrivial radially symmetric solution.

**Remark 1.2.** Condition \(0 < \lambda \leq \mu\) shows that the potential \(\mu/|x|^2 - \lambda\) is positive in \(B_1\). Thus the Brézis-Nirenberg method (cf. [1]) cannot be used.

**Theorem 1.3.** If \(\mu > 0\) and \(u \in H^1_0(B_1)\) is a solution of (1.1), then there are \(C_1, C_2 > 0\) and \(\delta > 0\) such that \(C_2|x|^\alpha \geq u(x) \geq C_1|x|^\alpha\), for \(x \in B_\delta\), where \(\alpha = (1/2)(\sqrt{(N - 2)^2 + 4\mu^2 - (N - 2)}) > 0\).

**Remark 1.4.** One can easily deduce that if \(u \in H^1_0(B_1)\) is a solution of (1.1), then \(u \in C^2(B_1 \setminus \{\theta\})\) and \(u > 0\) in \(B_1 \setminus \{\theta\}\). Theorem 1.3 shows that \(u(\theta) = 0\). It is greatly different from the case of \(\mu \leq 0\) (see [6]).
2. Proof of Theorem 1.1

Lemma 2.1. Every radially symmetric nonnegative solution $u$ of the equation

$$-\Delta u + \frac{\mu}{|x|^2} u(x) = u^{2^* - 1}(x), \quad u \in \mathcal{D}^{1,2}(\mathbb{R}^N),$$  \tag{2.1}$$

can be represented by $u(x) = \rho^{(N-2)/2} U(\rho x)$ for some positive number $\rho$, where

$$U(x) = \frac{C_0 |x|^\tau}{(1 + |x|^{4\tau/(N-2)})^{(N-2)/2}},$$  \tag{2.2}$$

$$\tau = \sqrt{((N-2)/2)^2 + \mu},$$

and $C_0$ is a constant.

Proof. Let \( t = -\ln |x|, \ \theta = x/|x|, \) and \( v(t, \theta) := e^{-(N-2)/2}t} u(e^{-t} \theta). \) Then by \[3\], we know that $v$ satisfies the equation

$$-v_{tt} - \Delta \theta v + \tau^2 v = v^{2^* - 1} \quad \text{in} \ \mathbb{R} \times S^{N-1}. \tag{2.3}$$

Since $u$ is radially symmetric, $v$ depends only on $t$ and satisfies $-v_{tt} + \tau^2 v = v^{2^* - 1}, v > 0$ in $\mathbb{R}$. By \[3\], we know that the only positive solutions of the equation are translation of

$$v(t) = \left(\frac{\tau^2 2^*}{2}\right)^{1/(2^* - 1)} \left(\cosh \left(\frac{2^* - 2}{2} \tau t\right)\right)^{-2/(2^* - 2)}. \tag{2.4}$$

Thus, every radially symmetric nonnegative solution $u$ of (2.1) can be represented by $u(x) = \rho^{(N-2)/2} U(\rho x)$ for some positive number $\rho$. \hfill \square

Define $\mathcal{D}_r^{1,2}(\mathbb{R}^N) := \{ u \in \mathcal{D}^{1,2}(\mathbb{R}^N) \mid u \text{ is radially symmetric} \}$ and $H_0^{1,r}(B_1) := \{ u \in H_0^1(B_1) \mid u \text{ is radially symmetric} \}$. Let

$$S_{\mu} := \inf_{u \in \mathcal{D}_r^{1,2}(\mathbb{R}^N), \ u \neq 0} \frac{\int_{\mathbb{R}^N} |
abla u|^2 + \mu \int_{\mathbb{R}^N} (u^2/|x|^2)}{(\int_{\mathbb{R}^N} |u|^{2^*})^{2/2^*}}. \tag{2.5}$$

It follows from Lemma 2.1 that $S_{\mu} = (\int_{\mathbb{R}^N} |
abla U|^2 + \mu \int_{\mathbb{R}^N} (U^2/|x|^2))/((\int_{\mathbb{R}^N} U^{2^*})^{2/2^*}$. Let $\Sigma = \{ u \in H_0^{1,r}(B_1) \mid \|u\|_{2^*} = 1 \}$. For $u \in \Sigma$, define

$$S_{\lambda,\mu}(u) = \int_{B_1} |
abla u|^2 + \mu \int_{B_1} \frac{u^2}{|x|^2} - \lambda \int_{B_1} u^2. \tag{2.6}$$

Lemma 2.2. If $N = 3$ and $3/4 < \lambda \leq \mu$ or if $N \geq 4$ and $0 < \lambda \leq \mu$, then $S_{\lambda,\mu} := \inf_{u \in \Sigma} S_{\lambda,\mu}(u) < S_{\mu}$. 

Proof. Let \( \eta \in C^\infty_0 (\mathbb{R}^N) \) be a cut function which satisfies \( 0 \leq \eta(x) \leq 1, |\nabla \eta| \leq 2 \) in \( \mathbb{R}^N \), \( \eta(x) \equiv 1 \) in \( B_{1/2} \), and \( \eta(x) \equiv 0 \) in \( \mathbb{R}^N \setminus B_1 \). Let \( U_\rho(x) := \rho^{(N-2)/2} U(\rho x) \) and \( u_\rho(x) = \eta(x) U_\rho(x) \). By (2.2), we know that when \( |x| \) is big enough, there are constants \( C_1, C_2 > 0 \) such that

\[
|U(x)| \leq \frac{C_1}{|x|^\tau + N/2 - 1}, \quad |\nabla U(x)| \leq \frac{C_2}{|x|^\tau + N/2}, \tag{2.7}
\]

since

\[
\int_{B_1} |\nabla u_\rho|^2 = \int_{B_1} \eta^2 |\nabla u_\rho|^2 + \int_{B_1} u_\rho^2 |\nabla \eta|^2 + 2 \int_{B_1} u_\rho \cdot \eta \cdot \nabla u_\rho \cdot \nabla \eta \leq \int_{B_1} |\nabla u_\rho|^2 + 4 \int_{B_1 \setminus B_{1/2}} u_\rho^2 + 4 \left( \int_{B_1 \setminus B_{1/2}} u_\rho^2 \right)^{1/2} \left( \int_{B_1 \setminus B_{1/2}} |\nabla u_\rho|^2 \right)^{1/2} = \int_{\mathbb{R}^N} |\nabla U|^2 + \int_{\mathbb{R}^N \setminus B_p} |\nabla U|^2 + \frac{4}{\rho^2} \int_{B_p \setminus B_{\rho/2}} U^2 + \frac{4}{\rho^2} \left( \int_{B_p \setminus B_{\rho/2}} U \right)^{1/2} \left( \int_{B_p \setminus B_{\rho/2}} |\nabla U|^2 \right)^{1/2}. \tag{2.8}
\]

By (2.7), when \( N = 3 \) and \( 3/4 < \lambda \leq \mu \) or when \( N \geq 4 \) and \( 0 < \lambda \leq \mu \), for \( \rho \) big enough,

\[
\int_{B_p \setminus B_{\rho/2}} U^2 \leq \int_{B_p \setminus B_{\rho/2}} \frac{C_1}{|x|^{2r+N-2}} dx = \frac{C_3}{\rho^{2r-2}}, \quad \int_{\mathbb{R}^N \setminus B_p} |\nabla U|^2 \leq \int_{\mathbb{R}^N \setminus B_p} \frac{C_2}{|x|^{2r+N}} dx = \int_{0}^{+\infty} \frac{C_4}{r^{2r+1}} dr = \frac{C_4}{\rho^{2r}}, \tag{2.9}
\]

\[
\int_{B_1} |\nabla u_\rho|^2 \leq \int_{\mathbb{R}^N} |\nabla U|^2 + \frac{C_5}{\rho^{2r}}, \quad \int_{B_1} u_\rho^2 \leq \int_{\mathbb{R}^N} U^2 + \frac{C_6}{\rho^{2r}}, \quad \int_{B_1} |u_\rho|^2 \geq \int_{\mathbb{R}^N} U^2 - \frac{C_7}{\rho^{2r}}, \tag{2.10}
\]

\[
\int_{B_1} u_\rho^2 \geq \frac{C_8}{\rho^8}. \tag{2.11}
\]

When \( N = 3 \) and \( 3/4 < \lambda \leq \mu \) or when \( N \geq 4 \) and \( 0 < \lambda \leq \mu \), we have \( 2r > 2 \). Thus by (2.10) and (2.11), we get

\[
S_{\lambda, \mu} \frac{u_\rho}{|u_\rho|^2} \leq S_\mu - \frac{C_0}{\rho^2} + o \left( \frac{1}{\rho^2} \right), \quad \text{as } \rho \to \infty. \tag{2.12}
\]

It proves the lemma.

Proof of Theorem 1.1. By Lemma 2.2 and [10, Theorem 8.8], we deduce that \( S_{\lambda, \mu} \) can be achieved by some \( 0 \leq u \in H_{0,r}(B_1) \), then \( S_{\lambda, \mu}^{-1/(2r-2)} u \) is a nontrivial radially symmetric solution of (1.1).
3. Proof of Theorem 1.3

Let $E$ be the space which is the completion of $C_0^\infty(B_1)$ under the norm $\|u\|_E = (\int_{B_1} |x|^{2\alpha} |\nabla u|^2 \, dx)^{1/2}$.

**Lemma 3.1** (see [2]). For all $u \in C_0^\infty(\mathbb{R}^N) \,(N \geq 3)$,

$$\left(\int_{\mathbb{R}^N} |x|^{-bp} |u|^p \, dx\right)^{2/p} \leq C_{a,b} \int_{\mathbb{R}^N} |x|^{-2a} |\nabla u|^2 \, dx,$$

where $-\infty < a < (N-2)/2$, $a \leq b \leq a + 1$, and $p = 2N/(N - 2 + 2(b - a))$.

Choosing $a = -\alpha$, $p = 2$ and $2^*$, respectively, in (3.1), we get the following lemma.

**Lemma 3.2.** There is a constant $C > 0$ such that, for any $u \in C_0^\infty(\mathbb{R}^N)$,

$$\left(\int_{\mathbb{R}^N} |x|^{2^*a} |u|^{2^*} \, dx\right)^{2/2^*} \leq C \int_{\mathbb{R}^N} |x|^{2a} |\nabla u|^2 \, dx,$$

$$\int_{\mathbb{R}^N} |x|^{2a-2} |u|^2 \, dx \leq C \int_{\mathbb{R}^N} |x|^{2a} |\nabla u|^2 \, dx.$$

**Proof of Theorem 1.3.** If $v \in H_0^1(B_1)$ is a solution of (1.1), then by the standard regularity theory, one can easily deduce that $v \in C^2(B_1 \setminus \{\theta\})$. Let $u(x) = |x|^{-\alpha} v(x)$ (this kind of transform has been used in [9]). Direct calculation shows that, for any $x \in B_1 \setminus \{\theta\}$,

$$-\text{div} \left(|x|^{2^*a} \nabla u\right) = |x|^{2^*a} u^{2^*-1} + \lambda |x|^{2a} u.$$

Since $v \in E$, then by **Lemma 3.1** we know that $v$ is a weak solution of (3.3), that is, for any $\zeta \in C_0^\infty(B_1)$,

$$\int_{B_1} |x|^{2a} \nabla u \nabla \zeta = \int_{B_1} |x|^{2^*a} u^{2^*-1} \zeta + \int_{B_1} |x|^{2a} u \zeta.$$

For $t > 2$, $k > 0$, define

$$h(r) = \begin{cases} r^{t/2}, & 0 \leq r \leq k, \\ \frac{t}{2} k^{t/2-1} r + \left(1 - \frac{t}{2}\right) k^{t/2}, & r \geq k, \end{cases}$$

and $\phi(r) = \int_0^r |h'(s)|^2 \, ds$. It is easy to verify that there exists a constant $C > 0$ independent of $k$ such that

$$|r \phi(r)| \leq \frac{t^2}{4(t-1)} |h(r)|^2,$$

$$|\phi(r) - h(r)h'(r)| \leq C_t |h(r)h'(r)|,$$

where $C_t = (t-2)/2(t-1) < 1$. 

Let \(0 < r_2 < r_1 < 1\) and \(\eta \in C_0^\infty(B(\theta, r_1))\) satisfying \(0 \leq \eta \leq 1, \eta \equiv 1 \) in \(B(\theta, r_2), \eta \equiv 0 \) in \(\mathbb{R}^N \setminus B(\theta, r_1)\), and \(|\nabla \eta| \leq 2/(r_1 - r_2)\). Notice that \(\eta^2 \phi(u) \in E\), then

\[
\int_{B_1} |x|^{2\alpha} \nabla u \nabla (\eta^2 \phi(u)) = \int_{B_1} |x|^{2\alpha} \eta^2 (h'(u))^2 |\nabla u|^2 + 2 \int_{B_1} |x|^{2\alpha} \eta \phi(u) \nabla u \nabla \eta \\
= \int_{B_1} |x|^{2\alpha} \eta^2 |\nabla (h(u))|^2 + 2 \int_{B_1} |x|^{2\alpha} \eta \phi(u) \nabla u \nabla \eta.
\]

(3.8)

Since \(|\nabla (\eta h(u))|^2 = \eta^2 |\nabla (h(u))|^2 + h^2(u) |\nabla \eta|^2 + 2 \eta h(u) \nabla (h(u)) \nabla \eta\), by (3.7), we have

\[
\int_{B_1} |x|^{2\alpha} \nabla u \nabla (\eta^2 \phi(u)) = \int_{B_1} |x|^{2\alpha} |\nabla (\eta h(u))|^2 - \int_{B_1} |x|^{2\alpha} h^2(u) |\nabla \eta|^2 \\
- 2 \int_{B_1} |x|^{2\alpha} \eta h(u) h'(u) \nabla u \nabla \eta + 2 \int_{B_1} |x|^{2\alpha} \eta \phi(u) \nabla u \nabla \eta \\
\geq \int_{B_1} |x|^{2\alpha} |\nabla (\eta h(u))|^2 - \int_{B_1} |x|^{2\alpha} h^2(u) |\nabla \eta|^2 \\
- 2 \int_{B_1} |x|^{2\alpha} \eta |\phi(u) - h(u) h'(u)| |\nabla u \nabla \eta| \\
\geq \int_{B_1} |x|^{2\alpha} |\nabla (\eta h(u))|^2 - \int_{B_1} |x|^{2\alpha} h^2(u) |\nabla \eta|^2 \\
- 2 C_i \int_{B_1} |x|^{2\alpha} |\eta h(u) \nabla (h(u)) \nabla \eta|.
\]

Since

\[
\int_{B_1} |x|^{2\alpha} |\eta h(u) \nabla (h(u)) \nabla \eta| = \int_{B_1} |x|^{2\alpha} \left| \left( \nabla (\eta h(u)) - h(u) \nabla \eta \right) \nabla \eta \right| |h(u)| \\
\leq \int_{B_1} |x|^{2\alpha} |h(u) \nabla (\eta h(u)) \nabla \eta| + \int_{B_1} |x|^{2\alpha} |h(u)|^2 |\nabla \eta|^2 \\
\leq \frac{1}{2} \int_{B_1} |x|^{2\alpha} h^2(u) |\nabla \eta|^2 + \frac{1}{2} \int_{B_1} |x|^{2\alpha} \left| \nabla (\eta h(u)) \right|^2 \\
+ \int_{B_1} |x|^{2\alpha} |h(u)|^2 |\nabla \eta|^2,
\]

(3.10)

and by (3.9), we deduce that

\[
\int_{B_1} |x|^{2\alpha} \nabla u \nabla (\eta^2 \phi(u)) \\
\geq \int_{B_1} |x|^{2\alpha} |\nabla (\eta h(u))|^2 - \int_{B_1} |x|^{2\alpha} h^2(u) |\nabla \eta|^2 \\
- 2 C_i \left( \frac{1}{2} \int_{B_1} |x|^{2\alpha} h^2(u) |\nabla \eta|^2 + \frac{1}{2} \int_{B_1} |x|^{2\alpha} \left| \nabla (\eta h(u)) \right|^2 + \int_{B_1} |x|^{2\alpha} |h(u)|^2 |\nabla \eta|^2 \right)
\]
By Lemma 3.1, we get that
\[
\int_{B_1} |x|^{2a} |\nabla (\eta \mathbf{h}(u))|^2 - (1 + 3C_t) \int_{B_1} |x|^{2a} h^2(u) |\nabla \eta|^2,
\]
and
\[
\geq \frac{Ct}{2(t-1)} \left( \int_{B_1} |x|^{2a} |\eta \mathbf{h}(u)|^2 \right)^{2/2^*} - (1 + 3C_t) \int_{B_1} |x|^{2a} h^2(u) |\nabla \eta|^2.
\]
(3.11)

By (3.6), we have
\[
\int_{B_1} |x|^{2a} u^{2^*-1} \eta^2 \phi(u) + \int_{B_1} |x|^{2a} u \eta^2 \phi(u)
\]
\[
\leq \frac{t^2}{4(t-1)} \int_{B_1} |x|^{2a} |u|^{2^*-2} |\eta \mathbf{h}(u)|^2 + \frac{t^2}{4(t-1)} \int_{B_1} |x|^{2a} |\eta \mathbf{h}(u)|^2
\]
\[
\leq \frac{t^2}{4(t-1)} \left( \int_{\eta \neq 0} |x|^{2a} |u|^{2^*} \right)^{(2^*-2)/2^*} \left( \int_{B_1} |\eta \mathbf{h}(u)|^2 \right)^{2/2^*}
\]
\[
+ \frac{t^2}{4(t-1)} \int_{B_1} |x|^{2a} |\eta \mathbf{h}(u)|^2.
\]
(3.12)

Notice that \( u \) is a solution of (3.3), by (3.11) and (3.12) we have
\[
\left( \int_{B_1} |x|^{2a} |\eta \mathbf{h}(u)|^2 \right)^{2/2^*}
\]
\[
\leq \frac{t}{2C} \left( \int_{\eta \neq 0} |x|^{2a} |u|^{2^*} \right)^{(2^*-2)/2^*} \left( \int_{B_1} |x|^{2a} |\eta \mathbf{h}(u)|^2 \right)^{2/2^*}
\]
\[
+ \frac{2(1 + 3C_t)(t-1)}{Ct} \int_{B_1} |x|^{2a} h^2(u) |\nabla \eta|^2 + \frac{t}{2C} \int_{B_1} |x|^{2a} |\eta \mathbf{h}(u)|^2.
\]
(3.13)

Choose \( r_1 \) small enough such that \((t/2C)\int_{\eta \neq 0} |x|^{2a} |u|^{2^*} \right)^{(2^*-2)/2^*} < 1/2\). Notice that \( 2(1 + 3C_t)(t-1)/t < 8 \) (since \( 0 < C_t < 1 \) and \( t > 2 \)) and \( |\nabla \eta| < 2/(r_1 - r_2) \), from (3.13) we have
\[
\left( \int_{B(\theta_{r_1}, r_2)} |x|^{2a} |h(u)|^2 \right)^{2/2^*}
\]
\[
\leq \left( \frac{64}{C(r_1 - r_2)^2} + \frac{t}{C} \right) \int_{B(\theta, r_1)} |x|^{2a} h^2(u).
\]
(3.14)

Choosing \( 2(N - 2\alpha)/(N - 2 + 2\alpha) > t_0 > 2 \) and letting \( k \to \infty \) in (3.14), we get
\[
\left( \int_{B(\theta_{r_2})} |x|^{2a} |u|^{2^* t_0/2} \right)^{2/2^*}
\]
\[
\leq \left( \frac{64}{C(r_1 - r_2)^2} + \frac{t_0}{C} \right) \int_{B(\theta_{r_2})} |x|^{2a} |u|^{t_0}.
\]
(3.15)

By Lemma 3.1, we know that \((\int_{B_1} |x|^{2a} |u|^{t_0})^{2/2^*} \leq \int_{B_1} |x|^{2a} |\nabla u|^2 < \infty\). Combining (3.15), we get that
\[
\int_{B_1} |x|^{2a} |u|^{2^* t_0/2} < \infty.
\]
(3.16)
Since

\[ \int_{B_{1}} |x|^{2\alpha} \nabla u \nabla (\phi(u)) = \int_{B_{1}} |x|^{2\alpha} |\nabla (h(u))|^{2} \geq \left( \int_{B_{1}} |x|^{2\alpha} |h(u)|^{2} \right)^{2/2^{*}}, \]

\[ \int_{B_{1}} |x|^{2\alpha} u_{t} \phi(u) + \int_{B_{1}} |x|^{2\alpha} u \phi(u) \]

\[ \leq \frac{t^{2}}{4(t-1)} \int_{B_{1}} |x|^{2\alpha} |u|^{2^{*}-2} |h(u)|^{2} + \frac{t^{2}}{4(t-1)} \int_{B_{1}} |x|^{2\alpha} |h(u)|^{2} \]

\[ \leq \frac{t^{2}}{4(t-1)} \left( \int_{B_{1}} |x|^{2\alpha} |u|^{2^{*}t_{0}/2} \right)^{2(2^{*}-2)/2^{*}t_{0}} \left( \int_{B_{1}} |x|^{2\alpha} |h(u)|^{q} \right)^{2/q} \]

\[ + \frac{t^{2}}{4(t-1)} \left( \int_{B_{1}} |x|^{2\alpha} |u|^{2^{*}t_{0}/2} \right)^{2(2^{*}-2)/2^{*}t_{0}} \left( \int_{B_{1}} |x|^{2\alpha} |h(u)|^{q} \right)^{2/q}, \]

where \( q = 2 \cdot 2^{*}t_{0}/((t_{0} - 2)2^{*} + 4) \) and \( 2/q + 1/q' = 1 \), we can deduce that if \( \epsilon > 0 \) small enough and \( t_{0} \in (2, 2 + \epsilon) \), then \( (2\alpha - 2^{*}a/q')^{1/q'} > -2 \). Thus \( (\int_{B_{1}} |x|^{(2\alpha - 2^{*}a/q')^{1/q'}})^{1/q'} < \infty \). Let \( C' = (\int_{B_{1}} |x|^{2\alpha} |u|^{2^{*}t_{0}/2})^{2(2^{*}-2)/2^{*}t_{0}} + (\int_{B_{1}} |x|^{2\alpha} q^{1/q}) \), then by (3.17), we have

\[ \left( \int_{B_{1}} |x|^{2\alpha} |h(u)|^{2^{*}} \right)^{2/2^{*}} \leq \frac{C't^{2}}{4(t-1)} \left( \int_{B_{1}} |x|^{2\alpha} |h(u)|^{q} \right)^{2/q}. \]

Letting \( k \to \infty \), we get

\[ |u|_{2^{*}t_{0}/2^{*}, a} \leq \left( \frac{C't^{2}}{4(t-1)} \right)^{1/t} |u|_{qt/2^{*}a}, \]

where \( |u|_{t^{2}/2^{*}, a} := (\int_{B_{1}} |x|^{2\alpha} |u|^{t})^{1/t} \).

Choose \( t_{1} = (2^{*}/q)^{n}, n = 1, 2, \ldots \). Then by (3.19) we have

\[ |u|_{2^{*}t_{n}/2^{*}, a} \leq \prod_{i=1}^{n} \left( \frac{C't_{i}^{2}}{4(t_{i} - 1)} \right)^{1/t_{i}} |u|_{2^{*}t_{n}/2^{*}, a}. \]

Letting \( n \to \infty \), we deduce that \( u \in L^{\infty}(B_{1}) \). Thus there is \( C_{2} > 0 \) such that \( v(x) \leq C_{2} |x|^{a} \).

Since \( \text{div}(|x|^{2\alpha} \nabla u) \leq 0 \), by [4, Lemma 4.2], we have \( u(x) \geq C'' > 0 \) for \( x \in B_{\delta} \). So, there is \( C_{1} > 0 \) such that \( u(x) \geq C_{1} |x|^{a} \) for \( x \in B_{\delta} \). \( \square \)
An elliptic problem with critical exponent

References


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