PERIODIC SOLUTIONS OF A CLASS OF NON-AUTONOMOUS SECOND-ORDER DIFFERENTIAL INCLUSIONS SYSTEMS

DANIEL PAŞCA

Received 15 March 2001

Using an abstract framework due to Clarke (1999), we prove the existence of periodic solutions for second-order differential inclusions systems.

1. Introduction

Consider the second-order system

\[
\ddot{u}(t) = \nabla F(t, u(t)) \quad \text{a.e. } t \in [0, T],
\]

\[
u(0) = u(T) = 0,
\]

where \( T > 0 \) and \( F : [0, T] \times \mathbb{R}^n \to \mathbb{R} \) satisfies the following assumption:

(A) \( F(t, x) \) is measurable in \( t \) for each \( x \in \mathbb{R}^n \) and continuously differentiable in \( x \) for a.e. \( t \in [0, T] \), and there exist \( a \in C(\mathbb{R}^+, \mathbb{R}^+) \), \( b \in L^1(0, T; \mathbb{R}^+) \) such that

\[
\|F(t, x)\| \leq a(\|x\|)b(t),
\]

\[
\|\nabla F(t, x)\| \leq a(\|x\|)b(t),
\]

for all \( x \in \mathbb{R}^n \) and a.e. \( t \in [0, T] \).

Wu and Tang in [4] proved the existence of solutions for problem (1.1) when \( F = F_1 + F_2 \) and \( F_1, F_2 \) satisfy some assumptions. Now we will consider problem (1.1) in a more general sense. More precisely, our results represent the extensions to systems with discontinuity (we consider the generalized gradients unlike continuously gradient in classical results).
2. Main results

Consider the second-order differential inclusions systems

\[
\ddot{u}(t) \in \partial F(t, u(t)), \quad \text{a.e.}\ t \in [0, T],
\]

\[
u(0) - u(T) = \ddot{u}(0) - \ddot{u}(T) = 0,
\]

where \( T > 0 \), \( F : [0, T] \times \mathbb{R}^n \to \mathbb{R} \) and \( \partial \) denotes the Clarke subdifferential.

We suppose that \( F = F_1 + F_2 \) and \( F_1, F_2 \) satisfy the following assumption:

\((\lambda, \mu)\)-subconvex with \( \lambda > 1/2 \) and \( \mu < 2\lambda^2 \) for a.e. \( t \in [0, T] \);

\( \zeta \in \partial F_2(t, x) \implies \|\zeta\| \leq c_1 \|x\| + c_2 \),

for all \( x \in \mathbb{R}^n \) and a.e. \( t \in [0, T] \);

\[
\frac{1}{\|x\|^2} \left[ \int_0^T F_1(t, \lambda x) \, dt + \int_0^T F_2(t, x) \, dt \right] \to \infty \quad \text{as} \ \|x\| \to \infty.
\]

Then problem (2.1) has at least one solution which minimizes \( \phi \) on \( H_T \).

Remark 2.2. Theorem 2.1 generalizes [3, Theorem 1]. In fact, [3, Theorem 1] follows from Theorem 2.1 letting \( F_1 = 0 \).

Theorem 2.3. Assume that \( F = F_1 + F_2 \) where \( F_1, F_2 \) satisfy assumption \((\lambda, \mu)\)-subconvex for a.e. \( t \in [0, T] \), and there exists \( y \in L^1(0, T; \mathbb{R}) \), \( h \in L^1(0, T; \mathbb{R}^n) \) with \( \int_0^T h(t) \, dt = 0 \) such that

\[
F_3(t, x) \geq [h(t), x] + y(t),
\]

for all \( x \in \mathbb{R}^n \) and a.e. \( t \in [0, T] \);

\( \zeta \in \partial F_2(t, x) \implies \|\zeta\| \leq c_1 \),
for all $x \in \mathbb{R}^n$ and all $t \in [0, T]$, and
\[ \int_0^T F_2(t, x)dt \geq c_0. \quad (2.7) \]
for all $x \in \mathbb{R}^n$;

(vi) \[ \frac{1}{T} \int_0^T F_1(t, x)dt + \int_0^T F_2(t, x)dt \rightarrow \infty \quad \text{as} \quad \|x\| \rightarrow \infty. \quad (2.8) \]

Then problem (2.1) has at least one solution which minimizes $\varphi$ on $H^1_T$.

Theorem 2.4. Assume that $F = F_1 + F_2$, where $F_1, F_2$ satisfy assumption $(A')$ and the following conditions:

(vii) $F_1(t, x)$ is $(\lambda, \mu)$-subconvex for a.e. $t \in [0, T]$, and there exists $\gamma \in L^1(0, T; \mathbb{R})$, $h \in L^1(0, T; \mathbb{R}^n)$ with $\int_0^T h(t)dt = 0$ such that
\[ F_1(t, x) \geq \langle h(t), x \rangle + \gamma(t), \quad (2.9) \]
for all $x \in \mathbb{R}^n$ and a.e. $t \in [0, T]$;

(viii) there exist $c_1, c_2 > 0$ and $\alpha \in (0, 1)$ such that
\[ \zeta \in \partial F_2(t, x) \Rightarrow \|\zeta\| \leq c_1 \|x\|^\alpha + c_2, \quad (2.10) \]
for all $x \in \mathbb{R}^n$ and a.e. $t \in [0, T]$;

(ix) \[ \frac{1}{T} \int_0^T F_2(t, x)dt \rightarrow \infty \quad \text{as} \quad \|x\| \rightarrow \infty. \quad (2.11) \]

Then problem (2.1) has at least one solution which minimizes $\varphi$ on $H^1_T$.

3. Preliminary results

We introduce some functional spaces. Let $[0, T]$ be a fixed real interval $(0 < T < \infty)$ and $1 < p < \infty$. We denote by $W^1_T$ the Sobolev space of functions $u \in L^p(0, T; \mathbb{R}^n)$ having a weak derivative $\dot{u} \in L^p(0, T; \mathbb{R}^n)$. The norm over $W^1_T$ is defined by
\[ \|u\|_{W^1_T} = \left( \int_0^T |u(t)|^p dt + \int_0^T |\dot{u}(t)|^p dt \right)^{1/p}. \quad (3.1) \]

We denote by $H^1_T$ the Hilbert space $W^{1,2}_T$. We recall that
\[ \|u\|_{H^1_T} = \left( \int_0^T |u(t)|^2 dt \right)^{1/2}, \quad \|u\|_\infty = \max_{t \in [0, T]} |u(t)|. \quad (3.2) \]

For our aims, it is necessary to recall some very well-known results (for proof and details see [2]):
154 Periodic solutions

Proposition 3.1. If \( u \in W^{1,p}_T \) then
\[
\|u\|_{\infty} \leq c \|u\|_{W^{1,p}_T}.
\] (3.3)

If \( u \in W^{1,p}_T \) and \( \int_0^T u(t)dt = 0 \) then
\[
\|u\|_{\infty} \leq c \|\dot{u}\|_{L^p}.
\] (3.4)

If \( u \in H^1_T \) and \( \int_0^T u(t)dt = 0 \) then
\[
\|u\|_{L^2} \leq T \pi \|\dot{u}\|_{L^2} \text{ (Wirtinger’s inequality)},
\]
\[
\|u\|_{\infty} \leq \frac{T}{12} \|\dot{u}\|_{L^2} \text{ (Sobolev inequality)}.
\] (3.5)

Proposition 3.2. If the sequence \((u_k)\) converges weakly to \( u \) in \( W^{1,p}_T \), then
\( (u_k) \) converges uniformly to \( u \) on \([0,T]\).

Let \( X \) be a Banach space. Now, following [1], for each \( x,v \in X \), we define the generalized directional derivative at \( x \) in the direction \( v \) of \( f \in \text{Lip}_{\text{loc}}(X,\mathbb{R}) \) as
\[
f^0(x;v) = \limsup_{y \to x, \lambda \to 0} \frac{f(y + \lambda v) - f(y)}{\lambda}
\] (3.6)
and denote \( x \) by
\[
\partial f(x) = \{ x^* \in X^*: f^0(x;v) \geq \langle x^*,v \rangle, \forall v \in X \}
\] (3.7)
the generalized gradient of \( f \) at \( x \) (the Clarke subdifferential).

We recall the Lebourg’s mean value theorem (see [1, Theorem 2.3.7]). Let \( x \) and \( y \) be points in \( X \), and suppose that \( f \) is Lipschitz on an open set containing the line segment \([x,y]\). Then there exists a point \( u \) in \((x,y)\) such that
\[
f(y) - f(x) \in \partial f(u; y-x).
\] (3.8)

Clarke considered in [1] the following abstract framework:

- let \((T,\mathcal{F},\mu)\) be a positive complete measure space with \( \mu(T) < \infty \), and let \( T \) be a separable Banach space;

- let \( Z \) be a closed subspace of \( L^p(T,Y) \) (for some \( p \in [1,\infty) \)), where \( L^p(T,Y) \) is the space of \( p \)-integrable functions from \( T \) to \( Y \);

- we define a functional \( f \) on \( Z \) via
\[
f(x) = \int_T f_t(x(t))\mu(dt),
\] (3.9)
where \( f_t: Y \to R, (t \in T) \) is a given family of functions;
we suppose that for each \( y \) in \( Y \) the function \( t \to f_t(y) \) is measurable, and that \( x \) is a point at which \( f(x) \) is defined (finitely).

**Hypothesis 3.3.** There is a function \( k \) in \( L^q(T, \mathbb{R}) \), \((1/p + 1/q = 1)\) such that, for all \( t \in T \),
\[
|f_t(y_1) - f_t(y_2)| \leq k(t) \|y_1 - y_2\|_Y \quad \forall y_1, y_2 \in Y.
\] (3.10)

**Hypothesis 3.4.** Each function \( f_t \) is Lipschitz (of some rank) near each point of \( Y \), and for some constant \( c \), for all \( t \in T, y \in Y \), one has
\[
\zeta \in \partial f_t(y) \implies \|\zeta\|_Y \leq c\{1 + \|y\|^\alpha + c_2\}. \quad (3.11)
\]

Under the conditions described above Clarke proved (see [1, Theorem 2.7.5]):

**Theorem 3.5.** Under either of Hypotheses 3.3 or 3.4, \( f \) is uniformly Lipschitz on bounded subsets of \( Z \), and there is
\[
\partial f(x) \subset \int_T \partial f_t(x(t)) \mu(dt).
\] (3.12)

Further, if each \( f_t \) is regular at \( x(t) \) then \( f \) is regular at \( x \) and equality holds.

**Remark 3.6.** The function \( f \) is globally Lipschitz on \( Z \) when Hypothesis 3.3 holds.

Now we can prove the following result.

**Theorem 3.7.** Let \( F : [0, T] \times \mathbb{R}^n \to \mathbb{R} \) be such that \( F = F_1 + F_2 \) where \( F_1, F_2 \) are measurable in \( t \) for each \( x \in \mathbb{R}^n \), and there exist \( k \in L^2(0, T; \mathbb{R}), a \in C([0, \infty), \mathbb{R}), b \in L^1(0, T; \mathbb{R}^+), c_1, c_2 > 0, \) and \( \alpha \in (0, 1) \) such that
\[
|F_1(t, x_1) - F_1(t, x_2)| \leq k(t) \|x_1 - x_2\|,
\] (3.13)
\[
|F_2(t, x_1)| + a(|x_1|)b(t),
\] (3.14)
\[
\zeta \in \partial F_2(t, x) \implies \|\zeta\| \leq c_1 \|x\|^{\alpha} + c_2,
\] (3.15)
for all \( t \in [0, T] \) and all \( x, x_1, x_2 \in \mathbb{R}^n \). We suppose that \( L : [0, T] \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R} \) is given by \( L(t, x, y) = \frac{1}{2} \|y\|^2 + F(t, x) \).

Then, the functional \( f : Z \to \mathbb{R} \), where
\[
Z = \left\{ (u, v) \in L^2(0, T; \mathbb{R}) : u(t) = \int_0^t v(s)ds + c, c \in \mathbb{R}^n \right\}
\] (3.16)
given by \( f(u, v) = \int_0^T L(t, u(t), v(t))dt \), is uniformly Lipschitz on bounded subsets of \( Z \) and
\[
\partial f(u, v) \subset \int_0^T \left[ \partial F_1(t, u(t)) + \partial F_2(t, u(t)) \right] \times \|v(t)\| \, dt.
\] (3.17)
Periodic solutions

Proof. Let \( L_1(t, x, y) = F_1(t, x) \), \( L_2(t, x, y) = \left( \frac{1}{2} \right) \| y \|^2 + F_2(t, x) \), and \( f_1, f_2 : Z \to \mathbb{R} \) given by \( f_1(u, v) = \int_0^T L_1(t, u(t), v(t)) \, dt \), \( f_2(u, v) = \int_0^T L_2(t, u(t), v(t)) \, dt \). For \( f_1 \) we can apply Theorem 3.5 under Hypothesis 3.3, with the following cast of characters:

- \((T, \mathcal{H}, \mu) = [0, T] \) with Lebesgue measure, \( Y = \mathbb{R}^n \times \mathbb{R}^n \) is the Hilbert product space (hence is separable);
- \( p = 2 \) and \( Z = \left\{ (u, v) \in L^2(0, T; Y) : u(t) = \int_0^t v(s) \, ds + c, c \in \mathbb{R}^n \right\} \) (3.18) is a closed subspace of \( L^2(0, T; Y) \);
- \( f_1(t, x, y) = L_1(t, x, y) = F_1(t, x) \); in our assumptions it results that the integrand \( L_1(t, x, y) \) is measurable in \( t \) for a given element \((x, y)\) of \( Y \) and there exists \( k \in L^2(0, T; \mathbb{R}) \) such that

\[
\left\| L_1(t, x_1, y_1) - L_1(t, x_2, y_2) \right\| \
\leq k(t) \left( \| x_1 - x_2 \| + \| y_1 - y_2 \| \right) \leq k(t) \| (x_1, y_1) - (x_2, y_2) \|_Y,
\]

for all \( t \in [0, T] \) and all \((x_1, y_1), (x_2, y_2) \in Y \). Hence \( f_1 \) is uniformly Lipschitz on bounded subsets of \( Z \) and one has

\[
\partial f_1(u, v) \subset \int_0^T \partial L_1(t, u(t), v(t)) \, dt.
\]

For \( f_2 \) we can apply Theorem 3.5 under Hypothesis 3.4 with the same cast of characters, but now \( f_2(t, x, y) = L_2(t, x, y) = \left( \frac{1}{2} \right) \| y \|^2 + F_2(t, x) \). In our assumptions, it results that the integrand \( L_2(t, x, y) \) is measurable in \( t \) for a given element \((x, y)\) of \( Y \) and locally Lipschitz in \((x, y)\) for each \( t \in [0, T] \).

Proposition 2.3.15 in [1] implies

\[
\partial f_2(u, v) \subset \int_0^T \partial L_2(t, u(t), v(t)) \, dt.
\]

Using (3.15) and (3.21), if \( \zeta = (\zeta_1, \zeta_2) \in \partial f_2(t, x, y) \) then \( \zeta_1 \in \partial F_2(t, x) \) and \( \zeta_2 = y \), and hence

\[
\| \zeta \| = \| \zeta_1 \| + \| \zeta_2 \| \leq c_1 \| x \|^p + c_2 + \| y \| \leq \epsilon \| (x, y) \|,
\]

for each \( t \in [0, T] \). Hence \( f_2 \) is uniformly Lipschitz on bounded subsets of \( Z \) and one has

\[
\partial f_2(u, v) \subset \int_0^T \partial L_2(t, u(t), v(t)) \, dt.
\]
It follows that $f = f_1 + f_2$ is uniformly Lipschitz on the bounded subsets of $Z$.

Propositions 2.3.3 and 2.3.15 in [1] imply that
\[
\partial f (u,v) \subset \partial f_1 (u,v) + \partial f_2 (u,v)
\]
\[
\subset \int_0^T \left[ (\partial L_1(t,u(t),v(t)) \times \partial y L_1(t,u(t),v(t))) + (\partial x L_2(t,u(t),v(t)) \times \partial y L_2(t,u(t),v(t)))\right] dt
\]
\[
= \int_0^T (\partial F_1(t,u(t)) + \partial F_2(t,u(t))) \times \{v(t)\} dt.
\]

Moreover, Corollary 1 of Proposition 2.3.3 in [1] implies that, if at least one of the functions $F_1, F_2$ is strictly differentiable in $x$ for all $t \in [0, T]$ then
\[
\partial f (u,v) \subset \int_0^T \partial F_1(t,u(t)) \times \{v(t)\} dt.
\]

Remark 3.8. The interpretation of expression (3.25) is that if $(u_0, v_0)$ is an element of $Z$ (so that $v_0 = \dot{u}_0$) and if $\xi \in \partial f (u_0, v_0)$, we deduce the existence of a measurable function $(q(t), p(t))$ such that
\[
\xi (u, v) = \int_0^T \left[ (\partial f_1(t,u(t)) \times q(t)) + (\partial f_2(t,u(t)) \times p(t))\right] dt.
\]

and for any $(u,v)$ in $Z$, one has
\[
\langle \xi, (u,v) \rangle = \int_0^T \left[ (\dot{q}(t), u(t)) + (\dot{p}(t), v(t))\right] dt.
\]

In particular, if $\xi = 0$ (so that $u_0$ is a critical point for $\phi(u) = \int_0^T (1/2)\|\dot{u}(t)\|^2 + F(t, u(t)))dt$), it then follows easily that $\dot{q}(t) = \dot{p}(t)$ a.e., or taking into account (3.26)
\[
\dot{u}_0(t) \in \partial F_1(t,u_0(t)) \quad a.e. \text{ on } [0, T],
\]
so that $u_0$ satisfies the inclusions system (2.1).

Remark 3.9. Of course, if $F$ is continuously differentiable in $x$, then system (2.1) becomes system (1.1).
Proof of Theorem 2.1. From assumption (A′) it follows immediately that there exist \( a \in C(\mathbb{R}^+, \mathbb{R}^+) \) and \( b \in L^1(0, T; \mathbb{R}^+) \) such that

\[
|F_1(t, x)| \leq a(\|x\|)b(t),
\]

for all \( x \in \mathbb{R}^n \) and all \( t \in [0, T] \). Like in [4], we obtain

\[
F_1(t, x) \leq (2\alpha \|x\|^\beta + 1)a_0b(t),
\]

for all \( x \in \mathbb{R}^n \) and all \( t \in [0, T] \), where \( \beta < 2 \) and \( a_0 = \max_{0 \leq s \leq 1} a(s) \).

For \( u \in H^1_T \), let \( \bar{u} = (1/T)^{1/2} \int_0^T u(t) dt \) and \( \tilde{u} = u - \bar{u} \). From Lebourg’s mean value theorem it follows that for each \( t \in [0, T] \) there exist \( z(t) \) in \((\bar{u}, u(t))\) and \( \xi \in \partial F_2(t, z(t)) \) such that

\[
F_2(t, u(t)) - F_2(t, \bar{u}) = \langle \xi, \tilde{u}(t) \rangle.
\]

It follows from (2.3) and Sobolev’s inequality that

\[
\left| \int_0^T [F_2(t, u(t)) - F_2(t, \bar{u})] dt \right| \\
\leq \int_0^T \| F_2(t, u(t)) - F_2(t, \bar{u}) \| dt \\
\leq \int_0^T \left[ 2c_1(\|\tilde{u}\|^\mu + \|\hat{u}\|^\mu) + c_2 \|\hat{u}\| \right] \|\tilde{u}\| dt \\
\leq 2c_1 T \|\bar{u}\|_\infty \|\tilde{u}\|^\mu + 2c_1 T \|\bar{u}\|_\infty^{\mu+1} + c_2 T \|\bar{u}\|_\infty \\
\leq \frac{3}{4} \|\bar{u}\|^2_\infty + \frac{T}{4} c_3 \|\tilde{u}\|^\mu + 2c_1 T \|\bar{u}\|_\infty^{\mu+1} + c_2 T \|\bar{u}\|_\infty \\
\leq \frac{1}{4} \|\tilde{u}\|^2_\infty + C_1 \|\bar{u}\|^2_\infty + C_2 \|\bar{u}\|_\infty + C_3 \|\bar{u}\|_\infty \
\]

for all \( u \in H^1_T \) and some positive constants \( C_1, C_2, \) and \( C_3 \). Hence we have

\[
\psi(u) \geq \frac{1}{4} \int_0^T \|\tilde{u}(t)\|^2 dt + \frac{1}{2} \int_0^T F_1(t, \bar{u}) dt - \int_0^T F_2(t, -\bar{u}) dt \\
+ \int_0^T F_2(t, \bar{u}) dt + \int_0^T \left[ F_2(t, u(t)) - F_2(t, \bar{u}) \right] dt \\
\geq \frac{1}{4} \|\tilde{u}\|^2_\infty - C_1 \|\bar{u}\|^2_\infty + C_2 \|\bar{u}\|_\infty + C_3 \|\bar{u}\|_\infty - \int_0^T a_0 \beta(t) dt \\
+ \frac{1}{4} \int_0^T F_1(t, \bar{u}) dt + \int_0^T F_2(t, \bar{u}) dt
\]
\[
\geq \frac{1}{2} \|\dot{u}\|_2^2 + C_1 \|\bar{u}\|_2^2 - C_2 \|\dot{u}\|_2^2 - C_3 + \|\ddot{u}\|_2^2 - C_5
\]
\[
\geq \frac{1}{2} \|\dot{u}\|_2^2 + \frac{1}{2} \left( \int_0^T F_1(t, \dot{u}) dt + \int_0^T F_2(t, \dot{u}) dt \right) - C_3 \}
\]
for all \( u \in H^1_T \), which implies that \( \psi(u) \to \infty \) as \( \|u\| \to \infty \) by (2.4) because \( \alpha < 1 \), \( \beta < 2 \), and the norm \( \| \bar{u} \|_2^2 + \| \dot{u} \|_2^2 \) is an equivalent norm on \( H^1_T \). Now we write
\[
\psi(u) = \psi_1(u) + \psi_2(u)
\]
where
\[
\psi_1(u) = \frac{1}{2} \left( \int_0^T \| \dot{u}(t) \|_2^2 dt - \left[ \int_0^T F_1(t, \dot{u}) dt + \int_0^T F_2(t, \dot{u}) dt \right] \right) - C_3
\]
\[
\psi_2(u) = \int_0^T F(t, u(t)) dt
\]
(4.5)

The function \( \psi_1 \) is weakly lower semi-continuous (w.l.s.c.) on \( H^1_T \). From (i), (ii), and Theorem 3.5, taking into account Remark 3.6 and Proposition 3.2, it follows that \( \psi_2 \) is w.l.s.c. on \( H^1_T \). By [2, Theorem 1.1], it follows that \( \psi \) has a minimum \( u_0 \) on \( H^1_T \). Evidently, \( Z \simeq H^1_T \) and \( \psi(u) = f(u,v) \) for all \( (u,v) \in Z \).

From Theorem 3.7, it results that \( f \) is uniformly Lipschitz on bounded subsets of \( Z \), and therefore \( \psi \) possesses the same properties relative to \( H^1_T \). Proposition 2.3.2 in [1] implies that \( 0 \in \partial \psi(u_0) \) (so that \( u_0 \) is a critical point for \( \psi \)). Now from Theorem 3.7 and Remark 3.8 it follows that problem (2.1) has at least one solution \( u \in H^1_T \).

\[ \square \]

**Proof of Theorem 2.3.** Let \((u_k)\) be a minimizing sequence of \( \psi \). It follows from (iv), (v), Lebourg’s mean value theorem, and Sobolev inequality, that
\[
\psi(u_k) \geq \frac{1}{2} \|\dot{u}_k\|_2^2 + \int_0^T \|b(t), u_k(t)\|dt + \int_0^T \|y(t)\|dt
\]
\[
+ \int_0^T F_2(t, \dot{u}_k) dt - \int_0^T \|\dot{u}_k(t)\|dt
\]
\[
\geq \frac{1}{2} \|\dot{u}_k\|_2^2 - \|\bar{u}_k\|_\infty \int_0^T \|b(t)\|dt
\]
\[
+ \int_0^T \|y(t)\|dt - c_1 \|\dot{u}_k\|_\infty + c_0
\]
\[
\geq \frac{1}{2} \|\dot{u}_k\|_2^2 - c_2 \|\dot{u}_k\|_2^2 - c_3
\]
(4.6)
for all \( k \) and some constants \( c_2, c_3 \), which implies that \( \|\dot{u}_k\| \) is bounded. On the other hand, in a way similar to the proof of Theorem 2.1, one has
\[
\left| \int_0^T [F_2(t, u(t)) - F_2(t, \bar{u})] dt \right| \leq \frac{1}{2} \|\dot{u}_k\|_2^2 + C_1 \|\bar{u}\|_2^2
\]
(4.7)
for all \( k \) and some positive constant \( C_1 \), which implies that

\[
\psi(u_k) \geq \frac{1}{2} \|\dot{u}_k\|_L^2 + \frac{1}{2} \int_0^T F_1(t, \dot{u}_k) dt + \int_0^T F_2(t, \ddot{u}_k) dt + \int_0^T \left[ F_2(t, \dot{u}_k) - F_2(t, \ddot{u}_k) \right] dt \\
\geq \frac{1}{4} \|\dot{u}_k\|_L^2 + \frac{1}{2} \int_0^T F_1(t, \dot{u}_k) dt + \int_0^T F_2(t, \ddot{u}_k) dt. 
\]

(4.8)

for all \( k \) and some positive constant \( C_1 \). It follows from (vi) and the boundedness of \( (\bar{u}_k) \) that \( (\bar{u}_k) \) is bounded. Hence \( \psi \) has a bounded minimizing sequence \( (u_k) \).

This completes the proof. □

Proof of Theorem 2.4. From (vii), (3.26), and Sobolev’s inequality it follows that

\[
\psi(u) \geq \frac{1}{2} \|\dot{u}\|_L^2 + \frac{1}{2} \int_0^T |h(t), u(t)| dt + \int_0^T \gamma(t) dt \\
+ \int_0^T F_2(t, \dot{u}) dt + \int_0^T \left[ F_2(t, \dot{u}) - F_2(t, \ddot{u}) \right] dt \\
\geq \frac{1}{4} \|\dot{u}\|_L^2 + \frac{1}{2} \int_0^T |h(t)| dt + \int_0^T \gamma(t) dt \\
- C_1 \|\dot{u}\|_{L^p}^{p+1} - C_2 \|\ddot{u}\|_2 + \int_0^T F_2(t, \dot{u}) dt - C_3 \|\ddot{u}\|_{L^p}^{p+1} \\
\geq \frac{1}{4} \|\dot{u}\|_L^2 - C_1 \|\dot{u}\|_{L^p}^{p+1} - C_2 \|\ddot{u}\|_2 - C_4 \|\ddot{u}\|_{L^p}^{p+1} \\
+ \|\ddot{u}\|_{L^p}^{p+1} \int_0^T F_2(t, \dot{u}) dt - C_3, 
\]

(4.9)

for all \( u \in H^1_T \) and some positive constants \( C_1, C_3, \) and \( C_4 \). Now it follows like in the proof of Theorem 2.1 that \( \psi \) is coercive by (ix), which completes the proof. □

References


Daniel Păsăcă: Department of Mathematics, University of Oradea, Armatei Romaniei 5, 3700, Oradea, Romania
E-mail address: pasca@mathematik.uni-kl.de
Special Issue on
Decision Support for Intermodal Transport

Call for Papers

Intermodal transport refers to the movement of goods in a single loading unit which uses successive various modes of transport (road, rail, water) without handling the goods during mode transfers. Intermodal transport has become an important policy issue, mainly because it is considered to be one of the means to lower the congestion caused by single-mode road transport and to be more environmentally friendly than the single-mode road transport. Both considerations have been followed by an increase in attention toward intermodal freight transportation research.

Various intermodal freight transport decision problems are in demand of mathematical models of supporting them. As the intermodal transport system is more complex than a single-mode system, this fact offers interesting and challenging opportunities to modelers in applied mathematics. This special issue aims to fill in some gaps in the research agenda of decision-making in intermodal transport.

The mathematical models may be of the optimization type or of the evaluation type to gain an insight in intermodal operations. The mathematical models aim to support decisions on the strategic, tactical, and operational levels. The decision-makers belong to the various players in the intermodal transport world, namely, drayage operators, terminal operators, network operators, or intermodal operators.

Topics of relevance to this type of decision-making both in time horizon as in terms of operators are:

- Intermodal terminal design
- Infrastructure network configuration
- Location of terminals
- Cooperation between drayage companies
- Allocation of shippers/receivers to a terminal
- Pricing strategies
- Capacity levels of equipment and labour
- Operational routines and lay-out structure
- Redistribution of load units, railcars, barges, and so forth
- Scheduling of trips or jobs
- Allocation of capacity to jobs
- Loading orders
- Selection of routing and service

Before submission authors should carefully read over the journal’s Author Guidelines, which are located at http://www.hindawi.com/journals/jamds/guidelines.html. Prospective authors should submit an electronic copy of their complete manuscript through the journal Manuscript Tracking System at http://mts.hindawi.com/, according to the following timetable:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Manuscript Due</td>
<td>June 1, 2009</td>
</tr>
<tr>
<td>First Round of Reviews</td>
<td>September 1, 2009</td>
</tr>
<tr>
<td>Publication Date</td>
<td>December 1, 2009</td>
</tr>
</tbody>
</table>

Lead Guest Editor

Gerrit K. Janssens, Transportation Research Institute (IMOB), Hasselt University, Agoralaan, Building D, 3590 Diepenbeek (Hasselt), Belgium; Gerrit.Janssens@uhasselt.be

Guest Editor

Cathy Macharis, Department of Mathematics, Operational Research, Statistics and Information for Systems (MOSI), Transport and Logistics Research Group, Management School, Vrije Universiteit Brussel, Pleinlaan 2, 1050 Brussel, Belgium; Cathy.Macharis@vub.ac.be