ON THE PROJECTION CONSTANTS OF SOME
TOPOLOGICAL SPACES AND SOME
APPLICATIONS

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Received 13 May 2001

We find a lower estimation for the projection constant of the projective tensor product $X \otimes \wedge Y$ and the injective tensor product $X \otimes \vee Y$, we apply this estimation on some previous results, and we also introduce a new concept of the projection constants of operators rather than that defined for Banach spaces.

1. Introduction

If $Y$ is a closed subspace of a Banach space $X$, then the relative projection constant of $Y$ in $X$ is defined by

$$\lambda(Y, X) := \inf \{ \| P \| : P \text{ is a linear projection from } X \text{ onto } Y \}. \quad (1.1)$$

And the absolute projection constant of $Y$ is defined by

$$\lambda(Y) := \sup \{ \lambda(Y, X) : X \text{ contains } Y \text{ as a closed subspace} \}. \quad (1.2)$$

It is well known that any Banach space $Y$ can be isometrically embedded into $l_\infty(\Gamma)$ for some index set $\Gamma$ (it is usually taken to be $U_Y^*$ where $Y^*$ denotes the dual space of $Y$ and $U_Y^*$ denotes the set $\{ f : f \in Y^* : \| f \| \leq 1 \}$) and that if $Y$ is complemented in $l_\infty(\Gamma)$, then it is complemented in every Banach space containing it as a closed subspace, that is, $Y$ is injective. We also know that for any such embedding the supremum in (1.2) is attained, that is, $\lambda(Y) = \lambda(\Gamma, l_\infty(\Gamma))$ (see [1, 4]). For each finite-dimensional space $Y_n$ with $\dim Y_n = n$, Kadets and Snobar [6] proved that $\lambda(Y_n) \leq \sqrt{n}$. König [7] showed that for each prime number $n$ the space $l_\infty^n$ contains an $n$-dimensional subspace $Y_n$ with projection constant

$$\lambda(Y_n) = \sqrt{n} - \left( \frac{1}{\sqrt{n}} - \frac{1}{n} \right). \quad (1.3)$$
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Konig and Lewis [9] verified the strict inequality \( \lambda(Y_n) < \sqrt{n} \) in case \( n \geq 2 \).

Lewis [14] showed that

\[
\lambda(Y_n) \leq \sqrt{n - \frac{1}{\sqrt{n} - 2}} + O\left(\frac{n^{-1/4}}{\sqrt{n}}\right)
\]

(1.4)

Konig and Tomczak-Jaegermann [11] also showed that there is a sequence \( \{X_n\}_{n \in \mathbb{N}} \) of Banach spaces \( X_n \) with \( \dim X_n = n \) such that

\[
\lim_{n \to \infty} \frac{\lambda(X_n)}{\sqrt{n}} = \frac{\sqrt{2}}{\pi}.
\]

(1.5)

In fact, it is shown in [9] that for each Banach space \( Y_n \) with dimension \( n \),

\[
\lambda(Y_n) \leq \sqrt{n - \frac{1}{\sqrt{n} - 2}} + O\left(\frac{n^{-1/4}}{\sqrt{n}}\right),
\]

in the real field,

\[
\lambda(Y_n) \leq \sqrt{n - \frac{1}{\sqrt{n} - 2}} + O\left(\frac{n^{-1/4}}{\sqrt{n}}\right),
\]

in the complex field.

(1.6)

(1.7)

The precise values of \( l_1^n, l_2^n \), and \( l_p^n, 1 < p < \infty, p \neq 2 \), have been calculated by Grünbaum [4], Rutovitz [15], Gordon [3], and Garling and Gordon [2]. In the case of \( 1 < p < 2 \), the improvement of these results was given by König, Schütt, and Tomczak-Jaegermann in [10], they showed that

\[
\lim_{n \to \infty} \frac{\lambda(l_p^n)}{\sqrt{p^n}} = \begin{cases} \frac{\sqrt{p^n}}{\sqrt{\pi}}, & \text{in the real field,} \\ \frac{\sqrt{p^n}}{2}, & \text{in the complex field.} \end{cases}
\]

Some other results are mentioned in [2, 3, 13, 15].

For finite codimensional subspaces, Garling and Gordon [2] showed that if \( Y \) is a finite codimensional subspace of the Banach space \( X \) with codimension \( n \), then for every \( \epsilon > 0 \) there exists a projection \( P \) from \( X \) onto \( Y \) with norm

\[
\|P\| \leq 1 + (1 + \epsilon)\sqrt{n}.
\]

(1.8)

2. Notations and basic definitions

The sets \( X, Y, Z, \) and \( E \) denote Banach spaces, \( X^* \) denotes the conjugate space of \( X \) and \( U_X \) denotes the unit ball of the space \( X \). Elements of \( X, Y, X^*, \) and \( Y^* \) will be denoted by \( x, y, \ldots, y, h, \ldots, g, k, \ldots \), respectively. The
Injective tensor product $X \otimes^\vee Y$ between the normed spaces $X$ and $Y$ is defined as the completion of the smallest cross norm on the space $X \otimes Y$ and the norm on the space $X \otimes Y$ is defined by

$$\left\| \sum_{i=1}^n x_i \otimes y_i \right\|_{X \otimes Y} = \sup \left\{ \left\| \sum_{i=1}^n f(x_i)g(y_i) \right\| : f \in U_X^*, g \in U_Y^* \right\}.$$  \hspace{1cm} (2.1)

where the supremum is taken over all functionals $f \in U_X^*$ and $g \in U_Y^*$.

The projective tensor product $X \otimes^\wedge Y$ between the normed spaces $X$ and $Y$ is defined as the completion of the largest cross norm on the space $X \otimes Y$ and the norm on $X \otimes Y$ is defined by

$$\left\| \sum_{i=1}^n x_i \otimes y_i \right\|_{X \otimes^\wedge Y} = \inf \left\{ \left\| \sum_{j=1}^m u_j \otimes v_j \right\| : \sum_{j=1}^m \left\| u_j \right\| \left\| v_j \right\| \leq \left\| \sum_{i=1}^n x_i \otimes y_i \right\| \right\}.$$  \hspace{1cm} (2.2)

where the infimum is taken over all equivalent representations $\sum_{j=1}^m u_j \otimes v_j \in X \otimes Y$ of $\sum_{i=1}^n x_i \otimes y_i$ (see [5]).

If $X$ is a Banach space on which every linear bounded operator from $X$ into any Banach space $Y$ is nuclear (this is the case in all finite-dimensional Banach spaces $X$), then for any Banach space $Y$ the space $X \otimes^\vee Y$ is isomorphically isometric to $X \otimes^\wedge Y$ (see [16]).

The set $\Omega = \{(f,g) : f \in U_X^*, g \in U_Y^*\} = U_X^* \times U_Y^*.$

We start with the following two lemmas.

**Lemma 2.1.** For Banach spaces $X$ and $Y$ there is a norm one projection from $l_\infty(U_X^*) \otimes^\vee l_\infty(U_Y^*)$ onto $l_\infty(\Omega)$.

**Proof.** Since the space $l_\infty(\Omega)$ has the 1-extension property, it is sufficient to show that $l_\infty(\Omega)$ can be isometrically embedded in the space $l_\infty(U_X^*) \otimes^\vee l_\infty(U_Y^*)$. In fact, every nonzero element $0 \neq \tilde{F} = [\tilde{F}((f,g)) : f \in U_X^*, g \in U_Y^*]$ in the space $l_\infty(\Omega)$ (note that the norm in this Banach space is given by $\left\| \tilde{F} \right\|_{l_\infty(\Omega)} = \sup_{f \in U_X^*} \sup_{g \in U_Y^*} \left| \tilde{F}((f,g)) \right|$) defines two scalar-valued functions $F \in l_\infty(U_X^*)$ and $G \in l_\infty(U_Y^*)$ by the following formulas:

$$F(f) = \sup_{g \in U_Y^*} \left| \tilde{F}((f,g)) \right|, \quad G(g) = \sup_{f \in U_X^*} \left| \tilde{F}((f,g)) \right|.$$  \hspace{1cm} (2.3)

Clearly the element $\tilde{F} = (1/\left\| \tilde{F} \right\|_{l_\infty(\Omega)}) \times (F \otimes G)$ is an element of the space $l_\infty(U_X^*) \otimes^\vee l_\infty(U_Y^*)$. Since both the injective and the projective tensor products are cross norms, $\left[ \tilde{F} \right]_{l_\infty(U_X^*) \otimes^\vee l_\infty(U_Y^*)} = \left[ \tilde{F} \right]_{l_\infty(\Omega)}$. The mapping $J$ defined by the formula $J(F) = \tilde{F}$ is the required isometric embedding. \hspace{1cm} □

**Lemma 2.2.** Let $X$ and $Y$ be two Banach spaces. Then $\lambda(X \otimes^\vee Y) = \lambda(X \otimes^\wedge Y, l_\infty(\Omega))$. 

Proof. It is also sufficient to show that the space $X \otimes Y$ can be isometrically embedded in $l_\infty(\Omega)$. In fact, every element $\hat{F} = \sum_{i=1}^\infty f_i \otimes g_i \in X \otimes Y$ defines a scalar-valued bounded function $\hat{F} \in l_\infty(\Omega)$ by the formula $\hat{F}(f, g) = \sum_{i=1}^\infty f_i g_i$. Using definition (2.1) for the injective tensor product, we have $\|\hat{F}\| = \|\hat{F}\|_{l_\infty(\Omega)}$. The mapping $i$ defined by the formula $i(F) = \hat{F}$ is the required isometric embedding. □

We have the following theorem.

**Theorem 2.3.** (1) If $Y_1$ and $Y_2$ are complemented subspaces of Banach spaces $X_1$ and $X_2$, respectively, then the injective (resp., projective) tensor product $Y_1 \otimes Y_2$ (resp., $Y_1 \otimes Y_2$) of the spaces $Y_1$ and $Y_2$ is complemented in the injective (resp., projective) tensor product $X_1 \otimes X_2$ (resp., $X_1 \otimes X_2$) and

$$\lambda \left( Y_1 \otimes Y_2, X_1 \otimes X_2 \right) \leq \lambda \left( Y_1, X_1 \right) \lambda \left( Y_2, X_2 \right).$$

(2.4)

(2) If $X$ and $Y$ are injective spaces, then the space $X \otimes Y$ is injective. Moreover,

$$\lambda \left( X \otimes Y \right) \leq \lambda(X) \lambda(Y).$$

(2.5)

**Proof.** Let $P_1$ and $P_2$ be any projections from $X_1$ onto $Y_1$ and from $X_2$ onto $Y_2$, respectively. Then the operator $P$ from the space $X_1 \otimes X_2$ onto the space $Y_1 \otimes Y_2$ (resp., from the space $X_1 \otimes X_2$ onto the space $Y_1 \otimes Y_2$) defined by

$$P \left( \sum_{i=1}^n x_i \otimes y_i \right) = \sum_{i=1}^n P_1(x_i) \otimes P_2(y_i)$$

(2.6)

is a projection and its norm $\|P\|$ is not exceeding $\|P_1\|\|P_2\|$. In fact, let $\sum_{i=1}^n x_i \otimes y_i$ be any element of the space $X_1 \otimes X_2$. Then, in the case of projective tensor product we have

$$\left\| P \left( \sum_{i=1}^n x_i \otimes y_i \right) \right\|_{T_1 \otimes T_2} \leq \sum_{i=1}^n \left\| P_1(x_i) \right\|_{T_1} \left\| P_2(y_i) \right\|_{T_2} \leq \|P_1\|\|P_2\| \sum_{i=1}^n \left\| x_i \right\| \left\| y_i \right\|.$$

(2.7)
for all equivalent representations \( \sum_{j=1}^{m} u_j \otimes v_j \) of \( \sum_{i=1}^{n} x_i \otimes y_i \). So
\[
\left\| P \left( \sum_{i=1}^{n} x_i \otimes y_i \right) \right\|_{Y_1 \otimes Y_2} \leq \| P_1 \| \| P_2 \| \left\| \sum_{i=1}^{n} x_i \otimes y_i \right\|_{X_1 \otimes X_2}.
\] (2.8)

And in the case of injective tensor product we have
\[
\left\| P \left( \sum_{i=1}^{n} x_i \otimes y_i \right) \right\|_{Y_1 \otimes Y_2} = \sup \left\{ \left\| \sum_{i=1}^{n} f(P_1(x_i)) \otimes g(P_2(y_i)) \right\| : f \in U_{Y_1}, g \in U_{Y_2} \right\}.
\] (2.9)

Thus in both cases, \( \| P \| \leq \| P_1 \| \| P_2 \| \). Taking the infimum of each side with respect to all such \( P_1 \) and \( P_2 \), we get inequality (2.4). To prove inequality (2.5), we apply inequality (2.4) and get in particular
\[
\lambda(X \otimes Y, l_\infty(U_{X^*} \otimes l_\infty(U_{Y^*})) \geq \lambda(X, l_\infty(U_{X^*})) \lambda(Y, l_\infty(U_{Y^*})).
\] (2.10)

Using Lemma 2.2 and definition (1.2), we get \( \lambda(X \otimes Y, l_\infty(U_{X^*} \otimes l_\infty(U_{Y^*}))) = \lambda(X \otimes Y). \) We claim that the sign \( \geq \) is an equal sign. In fact, if \( P \) is any projection from \( l_\infty(U_{X^*} \otimes l_\infty(U_{Y^*})) \) onto \( X \otimes Y \) and \( J \) is the embedding given in Lemma 2.1, then \( P = PJ \) is a projection from \( l_\infty(U_{X^*}) \) onto \( X \otimes Y \) with \( \| P \| \leq \| P \| \). This is the sufficient condition for the two infimum
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\[ \lambda(X \otimes Y, l_{\infty}(\Omega_1)) \] 
and 

\[ \lambda(X \otimes Y, l_{\infty}(U_X \ast \otimes \ast l_{\infty}(U_Y))) \] 
to be equal. Therefore 

\[ \lambda(X \otimes Y) = \lambda(X \otimes Y, l_{\infty}(U_X \ast \otimes \ast U_Y)). \] (2.11)

Using inequality (2.10), we get (2.5).

\[ \square \]

**Remark 2.4.** Since 

\[ \lambda(l_{\infty}(\Gamma_1)) = 1 \] 
for any index set \( \Gamma_1 \), we conclude that 

\[ \lambda(l_{\infty}(\Gamma_1) \otimes (\vee \text{ or } \wedge) l_{\infty}(\Lambda_1), X \otimes (\vee \text{ or } \wedge) Y) = 1 \] 
for every 

\[ X \supset l_{\infty}(\Gamma_1) \text{ and } Y \supset l_{\infty}(\Lambda_1). \]

We have the following two corollaries.

**Corollary 2.5.** For any finite sequence \( \{X_i\}_{i=1}^n \) of Banach spaces with complemented subspaces \( \{Y_i\}_{i=1}^n \), the relative projection constant of the injective (resp., projective) tensor product \( \bigotimes_{i=1}^n Y_i \) of the spaces \( Y_i \) in the space \( \bigotimes_{i=1}^n X_i \) satisfies

\[ \lambda \left( \bigotimes_{i=1}^n Y_i, \bigotimes_{i=1}^n X_i \right) \leq \prod_{i=1}^n \lambda(Y_i, X_i). \] (2.12)

**Corollary 2.6.** Let \( \{Y_i\}_{i=1}^n \) be a finite sequence of finite-dimensional Banach spaces. Then the relation between the absolute projection constant of the injective (or projective) tensor product \( \bigotimes_{i=1}^n Y_i \) and the direct sum \( \sum_{i=1}^n \bigoplus Y_i \) (with the supremum norm) is as follows:

\[ \lambda \left( \bigotimes_{i=1}^n Y_i \right) \leq \lambda \left( \sum_{i=1}^n \bigoplus Y_i \right)^n. \] (2.13)

**Proof.** In fact, the proof is a combination of Corollary 2.5 and the results of [3, Theorem 4].

\[ \square \]

### 3. Applications

In this section, using Theorem 2.3, we obtain new results.

1. For finite-dimensional Banach spaces \( X \) and \( Y \) with dimensions \( n \) and \( m \), respectively, we have

\[ \lambda(X \otimes Y) \leq \sqrt{nm} - \frac{1}{\sqrt{nm}} + O\left((nm)^{-3/4}\right) \]

\[ - \left\{ \left( \sqrt{m} - \frac{1}{\sqrt{m}} \right) \left( \frac{1}{\sqrt{n}} - O\left((m)^{-3/4}\right) \right) \right\} + \left( \sqrt{n} - \frac{1}{\sqrt{n}} \right) \left( \frac{1}{\sqrt{m}} - O\left((n)^{-3/4}\right) \right) \]. (3.1)
in the real field

\[ \lambda(X \otimes Y) \leq \sqrt{\frac{1}{2\sqrt{2}}} + O(\sqrt{m^{-3/4}}) \]

\[ - \left( \sqrt{\frac{1}{2\sqrt{2}}} \right) \left( \frac{1}{2\sqrt{2}} - O(m^{-3/4}) \right) \]  

\[ + \left( \sqrt{\frac{1}{2\sqrt{2}}} \right) \left( \frac{1}{2\sqrt{2}} - O(m^{-3/4}) \right) \}

in the complex field. Compare this result with the result in (1.6).

(2) For any positive integer \( m \) (not necessarily prime) with a prime factorization \( m = \prod_{i=1}^{n} q_i \) where the numbers \( q_i \) are distinct prime numbers, the space \( \bigotimes_{i=1}^{n} l_{q_i}^\infty \) contains a subspace \( Y \) of dimension \( m \) with

\[ \lambda(Y) \leq \sqrt{\prod_{i=1}^{n} q_i} - \sqrt{\prod_{i=1}^{n} q_i - 1} - C(m), \]

where \( C(m) \) is a positive number depending on \( m \) (in case of \( m = q_1 q_2 \), \( C(m) = \left( \frac{1}{\sqrt{q_1} - \frac{1}{\sqrt{q_2}}} \right) \left( \frac{1}{\sqrt{q_2} - 1} \right) \)). Comparing this result with (1.3), we mention that the \( m^2 \)-dimension of the space \( \bigotimes_{i=1}^{n} l_{q_i}^\infty \) is not a square of a prime number, so it gives a new subspace \( Y \) with a new projection constant.

(3) For numbers \( p, q \) with \( 1 \leq p, q \leq 2 \), we have

\[ \lim_{n,m \to \infty} \frac{\lambda\left( l_{p/q}^\infty \right)}{\sqrt{2m}} \leq \begin{cases} \frac{2}{\pi} & \text{in the real field,} \\ \frac{\pi}{2} & \text{in the complex field.} \end{cases} \]

4. The projection constants of operators

Now we start with our basic definitions of the projection constants of operators.

**Definition 4.1.** (1) A linear bounded operator \( A \) from a Banach space \( X \) into a Banach space \( Y \) is said to be left complemented with respect to a Banach space \( Z \) (\( Z \) contains \( Y \) as a closed subspace) if and only if there exists a linear bounded operator \( B \) from \( Z \) into \( X \) such that the composition \( AB \) is a projection from \( Z \) onto \( Y \). In this case \( Z \) is said to be a left complementation of \( A \).

If \( P_Z(A) \) denotes the convex set of all operators \( B \) from \( Z \) into \( X \) such that the composition \( AB \) is a projection, then

(2) the left relative projection constant of the operator \( A \) with respect to the space \( Z \) is defined as

\[ \lambda_l(A, Z) := \inf \left\{ \| AB \| : B \in P_Z(A) \right\}. \]
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(3) And the left absolute projection constant of $A$ is defined as

$$\lambda_l(A) := \sup \{ \lambda_l(A, Z) : Z \text{ is a left complementation of the operator } A \}. \quad (4.2)$$

We define the same analogy from the right.

Remark 4.2. We notice the following.

1. From the definition of $\lambda_l(A, Z)$, the infimum in (4.1) is taken only with respect to the projections that are factored (through $X$) into two operators one of them is $A$ and the other is an operator from $Z$ into $X$, so $1 \leq \lambda(Y, Z) \leq \lambda_l(A, Z)$ for every left complementation $Z$ of $A$.

2. If $A$ is a projection from $X$ onto $Y$, then $A$ is left complemented with respect to $Y$. In fact $AJ$ is a projection for any embedding $J$ from $Y$ into $X$.

3. If $I_Y$ is the identity operator on $Y$ and $X$ contains $Y$ as a complemented subspace, then $I_Y P = P$ for every projection $P$ from $X$ onto $Y$ and hence $I_Y$ is left complemented with respect to $X$. Moreover, $\lambda_l(I_Y, X) = \lambda(Y, X)$, that is, the relative projection constant of the identity operator on the space $Y$ with respect to the space $X$ is the relative projection constant of the space $Y$ in the space $X$.

4. If $Z$ is a left complementation of the linear bounded operator $A : X \to Y$, then $Y$ is complemented in $Z$ and the operator $A$ is onto.

5. If $Z$ is a separable or reflexive Banach space and $X$ is a Banach space, then for any index set $\Gamma$ the space $Z$ is not a right complementation of any linear bounded operator from $l_\infty(\Gamma)$ into $X$. In particular, if $X$ is a Banach space, then for any index set $\Gamma$, the space $l_\infty(\Gamma)$ is not a left complementation of any linear bounded operator from $X$ into the space $c_0$.

The following lemma is parallel to that lemma mentioned in [8] for Banach spaces and we omit the proof since the proof is nearly similar.

**Lemma 4.3.** Let $\Gamma$ be an index set such that $\Gamma$ is isometrically embedded into $l_\infty(\Gamma)$ and let $A$ be a linear bounded operator from $X$ onto $Y$ such that $\iota_{\infty}(\Gamma)$ is one of its left complementation. Then for a given $B \in P_{l_\infty(\Gamma)}(A)$,

1. For all Banach spaces $E, Z, E \subseteq Z$ and every linear bounded operator $T$ from $E$ into $Y$ there is an operator $T$ from $Z$ into $Y$ extending the operator $T$ with $\|T\| \leq \|AB\|\|T\|$, that is, the space $Y$ has $\|AB\|$-extension property, and in particular, if $Z \supseteq X$, the operator $A$ has a linear extension $\hat{A}$ from $Z$ into $Y$ with $\|\hat{A}\| \leq \|AB\|\|A\|$. That is, the extension constant $c(A)$ of the operator $A$ defined by ($c(A) := \sup_{\gamma \in \Gamma} \inf \{\|\hat{A}\| : \hat{A} \text{ is an extension of } A, \hat{A} : Z \to Y\}$) satisfies $c(A) \leq \|AB\|\|A\|$.

2. For every Banach space $Z \supseteq Y$, there exists a projection $P$ from $Z$ onto $Y$ such that $\|P\| \leq \|AB\|$.

The following theorem is also parallel to that given in (1.3) for Banach spaces.
Theorem 4.4. Let $Y$ be isometrically embedded in $l_{\infty}(\Gamma)$ and let $A$ be a linear bounded operator from $X$ onto $Y$ such that $l_{\infty}(\Gamma)$ is a left complementation of $A$. Then $A$ is left complemented with respect to any other Banach space $Z$ containing $Y$ as a closed subspace. Moreover,

$$\lambda_l(A, Z) \leq \lambda_l(A, l_{\infty}(\Gamma))$$

(4.3)

for every Banach space $Z$ containing $Y$ as a closed subspace, that is, $\lambda_l(A)$ attains its supremum at $l_{\infty}(\Gamma)$. Therefore,

$$\lambda_l(A) = \lambda_l(A, l_{\infty}(\Gamma)), \quad c(A) \leq \|A\| \lambda_l(A).$$

(4.4)

References


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Space dynamics is a very general title that can accommodate a long list of activities. This kind of research started with the study of the motion of the stars and the planets back to the origin of astronomy, and nowadays it has a large list of topics. It is possible to make a division in two main categories: astronomy and astrodynamics. By astronomy, we can relate topics that deal with the motion of the planets, natural satellites, comets, and so forth. Many important topics of research nowadays are related to those subjects. By astrodynamics, we mean topics related to spaceflight dynamics. It means topics where a satellite, a rocket, or any kind of man-made object is travelling in space governed by the gravitational forces of celestial bodies and/or forces generated by propulsion systems that are available in those objects. Many topics are related to orbit determination, propagation, and orbital maneuvers related to those spacecrafts. Several other topics that are related to this subject are numerical methods, nonlinear dynamics, chaos, and control.

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