PERMANENCE AND GLOBAL STABILITY OF POSITIVE SOLUTIONS OF A NONAUTONOMOUS DISCRETE RATIO-DEPENDENT PREDATOR-PREY MODEL

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We first give sufficient conditions for the permanence of nonautonomous discrete ratio-dependent predator-prey model. By linearization of the model at positive solutions and construction of Lyapunov function, we also obtain some conditions which ensure that a positive solution of the model is stable and attracts all positive solutions.

1. Introduction

In the theoretical ecology, permanence and global stability of the population model are very important. There are extensive literature related to these topics for differential equation models (see [3, 6, 7, 8, 9, 12] and the references cited therein). Recently, there has been a tendency for some researchers in the field of difference equations to develop some new methods which are analogous to those used in the study of differential equations. (See, e.g., [1, 2, 4, 5, 10, 11] and the references therein.)

In [5], Fan and Wang considered the following discrete periodic ratio-dependent predator-prey model:

\begin{align}
x_1(k + 1) &= x_1(k) \exp \left\{ a(k) - b(k)x_1(k) - \frac{c(k)x_2(k)}{m(k)x_2(k) + x_1(k)} \right\}, \\
x_2(k + 1) &= x_2(k) \exp \left\{ -d(k) + \frac{f(k)x_1(k)}{m(k)x_2(k) + x_1(k)} \right\},
\end{align}

(1.1)

and establish sufficient conditions for the existence of a positive periodic solution of the periodic system (1.1). In this paper, we will establish sufficient conditions for the permanence of system (1.1) and also obtain some conditions which ensure that a positive solution of the model is stable and attracts all positive solutions.

First, we present two definitions.

**Definition 1.1.** System (1.1) is defined to be permanent if there are positive constants $M$ and $m$ such that each positive solution $\{x_1(k), x_2(k)\}$ of system (1.1) satisfies

\begin{align}
m &\leq \lim \inf_{k \to \infty} x_i(k) \leq \lim \sup_{k \to \infty} x_i(k) \leq M, \quad i = 1, 2.
\end{align}

(1.2)

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Definition 1.2. System (1.1) is defined to be globally asymptotically stable if a positive solution of system (1.1) is stable and this solution attracts all positive solutions.

Throughout this paper, we will assume that $a(k), b(k), c(k), d(k), m(k)$, and $f(k)$ are bounded nonnegative sequences, and use the following notations: for any bounded sequence $\{u(k)\}$,

$$u^M = \sup_{k \in \mathbb{N}} u(k), \quad u^L = \inf_{k \in \mathbb{N}} u(k).$$  \hspace{1cm} (1.3)

For biological reasons, we only consider solution $\{x_1(k), x_2(k)\}$, with $x_1(0) > 0$, $x_2(0) > 0$.

The organization of this paper is the following. In the next section, we establish the permanence of system (1.1). In Section 3, we obtain the sufficient conditions which ensure that a positive solution of system (1.1) is stable and attracts all positive solutions.

2. Permanence

In this section, we establish a permanence result for system (1.1).

Lemma 2.1. For every solution $\{x_1(k), x_2(k)\}$ of (1.1),

$$\lim_{k \to \infty} \sup_{k \to \infty} x_1(k) \leq B_1, \quad \lim_{k \to \infty} \sup_{k \to \infty} x_2(k) \leq B_2,$$  \hspace{1cm} (2.1)

where

$$B_1 = \max \left\{ \frac{a^M}{b^L}, \exp \left( \frac{a^M - 1}{b^L} \right) \right\}, \quad B_2 = \left\{ \frac{f^M B_1}{m^L d^L}, \frac{f^M B_1}{m^L d^L} \exp \left\{ -d^L + f^M \right\} \right\}. \hspace{1cm} (2.2)$$

Proof. Clearly, $x_1(k) > 0$ and $x_2(k) > 0$ for $k \geq 0$. We first prove that

$$\lim_{k \to \infty} \sup_{k \to \infty} x_1(k) \leq B_1.$$  \hspace{1cm} (2.3)

To prove (2.3), we first assume that there exists an $l_0 \in \mathbb{N}$ such that $x_1(l_0 + 1) \geq x_1(l_0)$. Then,

$$a(l_0) - b(l_0) x_1(l_0) - \frac{c(l_0) x_2(l_0)}{m(l_0) x_2(l_0) + x_1(l_0)} \geq 0.$$  \hspace{1cm} (2.4)

Hence,

$$x_1(l_0) \leq \frac{a(l_0)}{b(l_0)} \leq \frac{a^M}{b^L}.$$  \hspace{1cm} (2.5)

It follows that

$$x_1(l_0 + 1) = x_1(l_0) \exp \left\{ a(l_0) - b(l_0) x_1(l_0) - \frac{c(l_0) x_2(l_0)}{m(l_0) x_2(l_0) + x_1(l_0)} \right\} \leq x_1(l_0) \exp \left\{ a(l_0) - b(l_0) x_1(l_0) \right\} \leq x_1(l_0) \exp \left\{ a^M - b^L x_1(l_0) \right\} \leq \frac{\exp \left( a^M - 1 \right)}{b^L},$$  \hspace{1cm} (2.6)
here we used
\[
\max_{x \in \mathbb{R}} \{ x \exp(a - bx) \} = \frac{\exp(a - 1)}{b}, \quad \text{for } a, b > 0. \quad (2.7)
\]

We claim that
\[
x_1(k) \leq B_1, \quad \text{for } k \geq l_0. \quad (2.8)
\]

By way of contradiction, assume that there exists a \( p_0 > l_0 \) such that \( x_1(p_0) > B_1 \). Then \( p_0 \geq l_0 + 2 \). Let \( \tilde{p}_0 \geq l_0 + 2 \) be the smallest integer such that \( x_1(\tilde{p}_0) > B_1 \). Then \( x_1(\tilde{p}_0 - 1) < x_1(\tilde{p}_0) \). The above argument produces that \( x_1(\tilde{p}_0) \leq B_1 \), a contradiction. This proves the claim. Now, we assume that \( x_1(k + 1) < x_1(k) \) for all \( k \in \mathbb{N} \). In particular, \( \lim_{k \to \infty} x_1(k) \) exists, denoted by \( x_1 \). We claim that \( x_1 \leq aM/bL \). By way of contradiction, assume that \( aM/bL > x_1 \). Taking limit in the first equation in system (1.1) gives
\[
\lim_{k \to \infty} \left( a(k) - b(k)x_1(k) - \frac{c(k)x_2(k)}{m(k)x_2(k) + x_1(k)} \right) = 0, \quad (2.9)
\]
which is a contradiction since
\[
a(k) - b(k)x_1(k) - \frac{c(k)x_2(k)}{m(k)x_2(k) + x_1(k)} \leq a(k) - b(k)x_1(k) \leq aM - bLx_1 < 0, \quad \text{for } n \in \mathbb{N}. \quad (2.10)
\]
This proves the claim. Note that \( aM/bL \leq B_1 \). It follows that (2.3) holds.

Next, we prove that
\[
\limsup_{k \to \infty} x_2(k) \leq B_2. \quad (2.11)
\]

At first, we assume that there exists an \( n_0 \in \mathbb{N} \) such that \( x_2(n_0 + 1) \geq x_2(n_0) \). Then
\[
-d(n_0) + \frac{f(n_0)x_1(n_0)}{m(n_0)x_2(n_0) + x_1(n_0)} \geq 0. \quad (2.12)
\]

Hence,
\[
x_2(n_0) \leq \frac{f(n_0)x_1(n_0)}{m(n_0)d(n_0)} \leq \frac{fMB_1}{mLdL}. \quad (2.13)
\]

It follows that
\[
x_2(n_0 + 1) = x_2(n_0) \exp \left\{ -d(n_0) + \frac{f(n_0)x_1(n_0)}{m(n_0)x_2(n_0) + x_1(n_0)} \right\} \leq \frac{fMB_1}{mLdL} \exp \{ -dL + fM \}. \quad (2.14)
\]
We claim that $x_2(k) \leq B_2$ for $k \geq n_0$. By way of contradiction, assume that there exists a $q_0 > n_0$ such that $x_2(q_0) > B_2$. Then $\tilde{q}_0 \geq n_0 + 2$ be the smallest integer such that $x_2(\tilde{q}_0) > B_2$. Then $x_2(\tilde{q}_0 - 1) < x_2(\tilde{q}_0)$. The above argument produces that $x_2(\tilde{q}_0) \leq B_2$, a contradiction. This proves the claim. Now, we assume that $x_2(k + 1) < x_2(k)$ for all $k \in \mathbb{N}$. In particular, $\lim_{k \to \infty} x_2(k)$ exists, denoted by $\overline{x}_2$. We claim that $\overline{x}_2 \leq f^MB_1/mLd^L$. By way of contradiction, assume that $\overline{x}_2 > f^MB_1/mLd^L$. Taking limit in the second equation in system (1.1) gives

$$
\lim_{k \to \infty} \left( -d(k) + \frac{f(k)x_1(k)}{m(k)x_2(k) + x_1(k)} \right) = 0, \tag{2.15}
$$

which is a contradiction since

$$
-d(k) + \frac{f(k)x_1(k)}{m(k)x_2(k) + x_1(k)} \leq -d^L + \frac{f^MB_1}{mL\overline{x}_2} < 0. \tag{2.16}
$$

It follows that (2.11) holds. This completes the proof. \hfill \square

**Lemma 2.2.** Assume that

$$
a^L > \frac{c^M}{m}, \quad f^L > d^M. \tag{2.17}
$$

Then

$$
\liminf_{k \to \infty} x_1(k) \geq D_1, \quad \liminf_{k \to \infty} x_2(k) \geq D_2, \tag{2.18}
$$

where

$$
D_1 = \min \left\{ \frac{a^L - c^M/mL}{b^M} \exp \left\{ a^L - b^MB_1 - \frac{c^M}{mL} \right\}, \frac{a^L - c^M/mL}{b^M} \right\},
$$

$$
D_2 = \min \left\{ \frac{f^L - d^M}{m^Md^M} D_1, \frac{f^L - d^M}{m^Md^M} \exp \left\{ -d^M + \frac{f^LD_1}{m^MB_2 + D_1} \right\} \right\}. \tag{2.19}
$$

**Proof.** We first show that

$$
\liminf_{k \to \infty} x_1(k) \geq D_1. \tag{2.20}
$$

According to Lemma 2.1, there exists a $k^* \in \mathbb{N}$ such that

$$
x_1(k) \leq B_1 + \epsilon, \quad x_2(k) \leq B_2 + \epsilon, \quad \text{for } k \geq k^*. \tag{2.21}
$$
Firstly, we assume that there exists an \( l_0 \geq k^* \) such that \( x_1(l_0 + 1) \leq x_1(l_0) \). Note that, for \( k \geq l_0 \),

\[
x_1(k + 1) = x_1(k) \exp \left\{ a(k) - b(k)x_1(k) - \frac{c(k)x_2(k)}{m(k)x_2(k) + x_1(k)} \right\}
\]

\[
\geq x_1(k) \exp \left\{ a(k) - b(k)x_1(k) - \frac{c(k)}{m(k)} \right\}
\]

\[
\geq x_1(k) \exp \left\{ a^L - b^M x_1(k) - \frac{c^M}{m^L} \right\}.
\]

In particular, with \( k = l_0 \), we have

\[
a^L - b^M x_1(l_0) - \frac{c^M}{m^L} \leq 0,
\]

which implies that

\[
x_1(l_0) \geq \frac{a^L - c^M/m^L}{b^M}.
\]

Then

\[
x_1(l_0 + 1) \geq \frac{a^L - c^M/m^L}{b^M} \exp \left\{ a^L - b^M(B_1 + \epsilon) - \frac{c^M}{m^L} \right\}.
\]

Let

\[
x_{1\epsilon} = \frac{a^L - c^M/m^L}{b^M} \exp \left\{ a^L - b^M(B_1 + \epsilon) - \frac{c^M}{m^L} \right\}.
\]

We claim that

\[
x_1(k) \leq x_{1\epsilon}, \quad \text{for } k \geq l_0.
\]

By way of contradiction, assume that there exists a \( p_0 \geq l_0 \) such that \( x_1(p_0) < x_{1\epsilon} \). Then \( p_0 \geq l_0 + 2 \). Let \( \tilde{p}_0 \geq l_0 + 2 \) be the smallest integer such that \( x_1(\tilde{p}_0) < x_{1\epsilon} \). Then \( x_1(\tilde{p}_0 - 1) > x_1(\tilde{p}_0) \). The above argument produces that \( x_1(\tilde{p}_0) < x_{1\epsilon} \), a contradiction. This proves the claim. Now, we assume that \( x_1(k + 1) > x_1(x) \) for all \( k \in \mathbb{N} \). In particular, \( \lim_{k \to \infty} x_1(k) \) exists, denoted by \( X_1 \). We claim that

\[
X_1 \geq \frac{a^L - c^M/m^L}{b^M}.
\]

By way of contradiction, assume that

\[
X_1 < \frac{a^L - c^M/m^L}{b^M}.
\]
Taking limit in the first equation in system (1.1) gives
\[
\lim_{k \to \infty} \left( a(k) - b(k)x_1(k) - \frac{c(k)x_2(k)}{m(k)x_2(k) + x_1(k)} \right) = 0, \quad (2.30)
\]
which is a contradiction since
\[
\lim_{k \to \infty} \left( a(k) - b(k)x_1(k) - \frac{c(k)x_2(k)}{m(k)x_2(k) + x_1(k)} \right) \geq a^L - b^M x_1 - \frac{c^M}{m^L} > 0. \quad (2.31)
\]
This proves the claim. It follows that (2.20) holds.

Next, we prove that
\[
\lim \inf_{k \to \infty} x_2(k) \geq D_2. \quad (2.32)
\]
At first, we assume that there exists an \( n_0 \in \mathbb{N} \) such that \( x_2(n_0 + 1) \geq x_2(n_0) \). Note that, for \( k \geq n_0 \),
\[
x_2(k + 1) = x_2(k) \exp \left\{ -d(k) + \frac{f(k)x_1(k)}{m(k)x_2(k) + x_1(k)} \right\} \geq x_2(k) \exp \left\{ -d(k) + \frac{f(k)D_1}{m(k)x_2(k) + D_1} \right\}. \quad (2.33)
\]
In particular, with \( k = n_0 \), we get
\[
-d(n_0) + \frac{f(n_0)D_1}{m(n_0)x_2(n_0) + D_1} \leq 0, \quad (2.34)
\]
which implies that
\[
x_2(n_0) \geq \frac{f^L - d^M}{m^M d^M} D_1. \quad (2.35)
\]
Then
\[
x_2(n_0 + 1) \geq \frac{f^L - d^M}{m^M d^M} D_1 \exp \left\{ -d^M + \frac{f^L D_1}{m^M (B_2 + \epsilon) + D_1} \right\}. \quad (2.36)
\]
Let
\[
x_{2e} = \frac{f^L - d^M}{m^M d^M} D_1 \exp \left\{ -d^M + \frac{f^L D_1}{m^M (B_2 + \epsilon) + D_1} \right\}. \quad (2.37)
\]
We claim that \( x_2(k) \geq x_{2e} \) for \( k \geq n_0 \). By way of contradiction, assume that there exists a \( q_0 \geq n_0 \) such that \( x_2(q_0) < x_{2e} \). Then \( q_0 \geq n_0 + 2 \). Let \( \tilde{q}_0 \geq n_0 + 2 \) be the smallest integer such that \( x_2(\tilde{q}_0) < x_{2e} \). Then \( x_2(\tilde{q}_0 - 1) > x_2(\tilde{q}_0) \). The above argument produces that \( x_2(\tilde{q}_0) \geq x_{2e} \), a contradiction. This proves the claim. Now, we assume that \( x_2(k + 1) < x_2(k) \) for all \( k \in \mathbb{N} \). In particular, \( \lim_{k \to \infty} x_2(k) \) exists, denoted by \( x_2 \). We claim that
\[
x_2 \geq \frac{f^L - d^M}{m^M d^M} D_1. \quad (2.38)
\]
By way of contradiction, assume that

$$x_2 < \frac{f_L - d_M}{m_M D_1}. \quad (2.39)$$

Taking limit in the second equation in system (1.1) gives

$$\lim_{k \to \infty} \left( -d(k) + \frac{f(k)x_1(k)}{m(k)x_2(k) + x_1(k)} \right) = 0, \quad (2.40)$$

which is a contradiction since

$$-d(k) + \frac{f(k)x_1(k)}{m(k)x_2(k) + x_1(k)} \geq -d_M + \frac{f_L D_1}{m_M x_2 + D_1} > 0. \quad (2.41)$$

It follows that (2.32) holds. This completes the proof. \[\square\]

Now, by Lemmas 2.1 and 2.2, we can easily obtain the following result.

**Theorem 2.3.** Assume that

$$a_L > c_M, \quad f_L > d_M. \quad (2.42)$$

Then system (1.1) is permanent.

### 3. Global stability

In this section, we derive sufficient conditions which guarantee that the positive solution of (1.1) is globally stable. Our strategy in the proof of the global stability of the positive solution of (1.1) is to construct suitable Lyapunov functions.

**Theorem 3.1.** In addition to the assumptions made in Theorem 2.3, assume further that

(i) there exist positive constant $\nu$ and positive constants $n_i$, $i = 1, 2$, such that

$$\min \left\{ n_1 b(k) - n_1 \frac{c(k)}{4m(k)D_1} - n_2 \frac{f(k)}{4D_1} \frac{D_1}{m(k)B_2 + B_1}, n_2 \frac{f(k)D_1}{(m(k)B_2 + B_1)^2} - n_1 \frac{c(k)}{4m(k)D_2} \right\} > \nu, \quad (3.1)$$

for all large $k$, where $D_1$ and $B_1$ are given in Lemmas 2.1 and 2.2,

(ii) $b(k)B_1 \leq 1$ and $f(k) \leq 4$ for all large $k$, where $B_1$ is given in Lemma 2.1.

Then system (1.1) is globally asymptotically stable, that is, a positive solution of (1.1) is stable and attracts all positive solutions.

**Proof.** Let $\{x_1^*(k), x_2^*(k)\}$ be a positive solution of (1.1). We prove below that it is uniformly asymptotically stable. To this end, we introduce the change of variables

$$u_1(k) = x_1(k) - x_1^*(k), \quad u_2(k) = x_2(k) - x_2^*(k). \quad (3.2)$$
System (1.1) is then transformed into

\[
\begin{align*}
    u_1(k+1) &= x_1(k) \exp \left\{ a(k) - b(k)x_1(k) - \frac{c(k)x_1^*(k)x_2^*(k)}{m(k)x_2^*(k) + x_1^*(k)} \right\} \\
    - x_1^*(k) \exp \left\{ a(k) - b(k)x_1^*(k) - \frac{c(k)x_1^*(k)x_2^*(k)}{a_1(k)x_2^*(k) + x_1^*(k)} \right\}, \\
    u_2(k+1) &= x_2(k) \exp \left\{ \frac{f(k)x_1(k)}{m(k)x_2(k) + x_1(k)} - d(k) \right\} \\
    - x_2^*(k) \exp \left\{ \frac{f(k)x_1^*(k)}{a_1(k)x_2^*(k) + x_1^*(k)} - d(k) \right\},
\end{align*}
\]

which, by Taylor formula, can be rewritten as

\[
\begin{align*}
    u_1(k+1) &= \exp \left\{ a(k) - b(k)x_1^*(k) - \frac{c(k)x_1^*(k)x_2^*(k)}{m(k)x_2^*(k) + x_1^*(k)} \right\} \\
    &\quad \times \left( 1 - b(k)x_1^*(k) + \frac{c(k)x_1^*(k)x_2^*(k)}{(m(k)x_2^*(k) + x_1^*(k))^2} \right) u_1(k) \\
    &\quad - \frac{c(k)x_1^*(k)}{(m(k)x_2^*(k) + x_1^*(k))^2} x_1^*(k) u_2(k) + f_1(k,u(k)), \\
    u_2(k+1) &= \exp \left\{ \frac{f(k)x_1^*(k)}{m(k)x_2^*(k) + x_1^*(k)} - d(k) \right\} \\
    &\quad \times \left( 1 - \frac{f(k)m(k)x_1^*(k)x_2^*(k)}{(m(k)x_2^*(k) + x_1^*(k))^2} \right) u_2(k) \\
    &\quad + \frac{f(k)m(k)x_2^*(k)}{(m(k)x_2^*(k) + x_1^*(k))^2} u_1(k) + f_2(k,u(k)),
\end{align*}
\]

where \(|f_i(k,u)|/\|u\|\) converges, uniformly with respect to \(k \in \mathbb{N}\), to zero as \(\|u\| \to 0\). In view of system (1.1), it follows from (3.4) that

\[
\begin{align*}
    u_1(k+1) &= x_1^*(k+1) \left( 1 - b(k)x_1^*(k) + \frac{c(k)x_1^*(k)x_2^*(k)}{(m(k)x_2^*(k) + x_1^*(k))^2} \right) u_1(k) \\
    &\quad - \frac{c(k)x_1^*(k)}{(m(k)x_2^*(k) + x_1^*(k))^2} u_2(k) + \frac{f_1(k,u(k))}{x_1^*(k)}, \\
    u_2(k+1) &= x_2^*(k+1) \left( 1 - \frac{f(k)m(k)x_1^*(k)x_2^*(k)}{(m(k)x_2^*(k) + x_1^*(k))^2} \right) u_2(k) \\
    &\quad + \frac{f(k)m(k)x_2^*(k)}{(m(k)x_2^*(k) + x_1^*(k))^2} u_1(k) + \frac{f_2(k,u(k))}{x_2^*(k)},
\end{align*}
\]
where \( |f_i(k,u)|/\|u\| \) converges, uniformly with respect to \( k \in \mathbb{N} \), to zero as \( \|u\| \to 0 \). We define the function \( V \) by

\[
V(u(k)) = n_1 \frac{u_1(k)}{x^*_1(k)} + n_2 \frac{u_2(k)}{x^*_2(k)},
\]

where \( n_j \) are positive constants given in (i). Calculating the difference of \( V \) along the solution of system (3.5) and using (ii), we obtain

\[
\Delta V \leq -\left( n_1 b(k) - n_1 \frac{c(k)}{4m(k)D_1} - n_2 \frac{f(k)}{4D_1} \right) x^*_1(k) \frac{u_1(k)}{x^*_1(k)}
- \left( n_2 \frac{f(k)D_1}{(m(k)B_2 + B_1)^2} - n_1 \frac{c(k)}{4m(k)D_2} \right) x^*_2(k) \frac{u_2(k)}{x^*_2(k)}
+ n_1 \frac{|f_1(k,u(k))|}{x^*_1(k)} + n_2 \frac{|f_1(k,u(k))|}{x^*_2(k)}, \quad \text{for large } k.
\]

Since \( |f_i(k,u)|/\|u\| \) converges uniformly to zero as \( \|u\| \to 0 \), it follows from condition (i) and Theorem 2.3 that there is a positive constant \( \gamma \) such that if \( k \) is sufficiently large and \( \|u(k)\| < \gamma \),

\[
\Delta V \leq -\frac{\nu \|u(k)\|}{2}.
\]

By [1], we see that the trivial solution of (3.5) is uniformly asymptotically stable, and so is the solution \( \{x^*_1(k),x^*_2(k)\} \) of (1.1). Note that the positive solution \( \{x_1(k),x_2(k)\} \) is chosen in an arbitrary way. Proceeding exactly as in [11], we conclude that the positive solution \( \{x^*_1(k),x^*_2(k)\} \) of (1.1) is globally stable. The proof is complete \( \square \)

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This subject has been extensively studied in the past years for one-, two-, and three-dimensional space. Additionally, such dynamical systems can exhibit a very important and still unexplained phenomenon, called as the Fermi acceleration phenomenon. Basically, the phenomenon of Fermi acceleration (FA) is a process in which a classical particle can acquire unbounded energy from collisions with a heavy moving wall. This phenomenon was originally proposed by Enrico Fermi in 1949 as a possible explanation of the origin of the large energies of the cosmic particles. His original model was then modified and considered under different approaches and using many versions. Moreover, applications of FA have been of a large broad interest in many different fields of science including plasma physics, astrophysics, atomic physics, optics, and time-dependent billiard problems and they are useful for controlling chaos in Engineering and dynamical systems exhibiting chaos (both conservative and dissipative chaos).

We intend to publish in this special issue papers reporting research on time-dependent billiards. The topic includes both conservative and dissipative dynamics. Papers discussing dynamical properties, statistical and mathematical results, stability investigation of the phase space structure, the phenomenon of Fermi acceleration, conditions for having suppression of Fermi acceleration, and computational and numerical methods for exploring these structures and applications are welcome.

To be acceptable for publication in the special issue of Mathematical Problems in Engineering, papers must make significant, original, and correct contributions to one or more of the topics above mentioned. Mathematical papers regarding the topics above are also welcome.

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