We establish a nonlinear real estate model based on cobweb theory, where the demand function and supply function are quadratic. The stability conditions of the equilibrium are discussed. We demonstrate that as some parameters varied, the stability of Nash equilibrium is lost through period-doubling bifurcation. The chaotic features are justified numerically via computing maximal Lyapunov exponents and sensitive dependence on initial conditions. The delayed feedback control (DFC) method is applied to control the chaos of system.

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1. Introduction

Cobweb models describe the price dynamics in a market of a nonstorable good that takes one time unit to produce [1]. In economic modeling, many examples of cobweb chaos have been demonstrated. Some of the most famous examples include [2–9]. Hommes [5] applies the concept of adaptive expectations in a cobweb model with a single producer to investigate the occurrence of strange and chaotic behavior. Finkenstädt [3] applied linear supply and nonlinear demand functions. Hommes [4] and Jensen and Urban [6] used linear demand functions with nonlinear supply equations. These findings indicate that the nonlinear cobweb model may explain various irregular fluctuations observed in real economic data. In this study, we go one step further to study the cobweb model with nonlinear demand and supply function. A possible source of such an evolutionary market dynamics is an interaction between government and real estate developer.

Traditional cobweb models usually describe a dynamic price adjustment in agricultural markets with a supply response lag [2]. Consider, for instance, the supply of housing. The time of housing construction guarantees a finite lag between the time the production
decision is made and the time the housing is ready for sale. The real estate developer’s decision about how many houses should be built and sale is usually based on current and past experience. This principle is the same as that of agricultural product. So it is feasible to introduce cobweb model into real estate market.

The present paper attempts to establish a nonlinear model for the real estate market, and introduce adjustment parameters of housing price and land price into the model, which can denote the game behavior of players. The system stability with the variation of parameters is analyzed. Numerical simulations verify the complexity of system evolution. Finally, time-delayed feedback control method is used to keep the system from chaos and bifurcation.

2. Nonlinear models for real estate market

In this paper we assume that all real estate developers in the market are belong to one benefit group and have a common benefit target. Usually the price $p$ is characterized by the nonlinear inverse demand function of $p = a - b\sqrt{Q}$, where $a$ and $b$ are positive constants, $a$ is the maximum price in the market, and $Q$ is the total quantity in the market. This kind of form has been used in other oligopoly models and in the experimental economics dealing with learning and expectations formation (see, e.g., [10–12]). The transformation of this formula is as follows:

$$
D_1(t) = b_0 - b_1 p_1(t) + b_2 p_1^2(t), \quad D_2(t) = c_0 - c_1 p_2(t) + c_2 p_2^2(t),
$$

(2.1)

where $b_0, b_1, b_2, c_0, c_1, c_2$ are positive constants, $p_1(t)$ is the land price at time period $t$, $p_2(t)$ is the housing price at time period $t$, $D_1(t)$ is the land demand at time period $t$, and $D_2(t)$ is the housing demand at time period $t$. Due to the law of demand that the slope of demand curve is negative, the prices $p_1(t)$ and $p_2(t)$ must, respectively, satisfy the inequalities: $2b_2 p_1(t) - b_1 < 0$ and $2c_2 p_2(t) - c_1 < 0$; $4b_2 b_0 - b_1^2 > 0$, $4c_2 c_0 - c_1^2 > 0$ must hold, thus the signs of demand equations in formula (2.1) are positive.

In this case, the land market and housing market are interrelated. Though the housing market does not directly affect land market, the land price impacts the housing supply which decreases with increasing land price. This rule is the same as that of hog and corn as stated by Waugh [13]. Real estate companies adjust the housing supply according to the relative policies and the situation of housing price and land price. The formula of supply can be supposed as follows:

$$
S_1(t) = e_0 + e_1 p_1(t) + e_2 p_1^2(t), \quad S_2(t) = d_0 + d_1 p_2(t) + d_2 p_2^2(t) - d_3 p_1(t),
$$

(2.2)

where $d_0, d_1, d_2, d_3, e_0, e_1, e_2$ are positive constants, $S_1(t)$ is the land supply at time period $t$, and $S_2(t)$ is the housing supply at time period $t$. Because $2e_2 p_1(t) + e_1 > 0$ and $2d_2 p_2(t) + d_1 > 0$, so we can affirm that the slope of supply curve is positive, and it is in accordance with the law of supply. Providers begin to supply the products only when

$$
p_1(t) > \frac{-e_1 + \sqrt{e_1^2 - 4e_2 e_0}}{2e_2}, \quad p_2(t) > \frac{-d_1 + \sqrt{d_1^2 - 4d_2 d_0}}{2d_2}.
$$

(2.3)
Define
\[ Z(p) = D(p) - S(p). \] (2.4)

\( Z(p) \) is excess demand function descending with price, which denotes the gap between demand and supply. When the price is low, excess demand exists and when the price is high, excess supply exists, thus \( p^* \) that satisfies the equation \( Z(p^*) = 0 \) is called equilibrium point.

Substituting (2.1) and (2.2) into (2.4), we obtain
\[
Z(p_1(t)) = b_0 - e_0 - (e_1 + b_1) p_1(t) + (b_2 - e_2) p_1^2(t) \\
Z(p_2(t)) = c_0 - d_0 - (d_1 + c_1) p_2(t) + (c_2 - d_2) p_2^2(t) + d_3 p_1(t), 
\] (2.5)

Since \( Z(p) \) follows the law of demand, the following conditions must hold:
\[
b_2 - e_2 > 0, \\
c_2 - d_2 > 0, \\
2(c_2 - d_2) p_2(t) - (d_1 + c_1) < 0, \\
2(b_2 - e_2) p_1(t) - (e_1 + b_1) < 0. 
\] (2.6)

\( \alpha_1 \) is the adjustment parameter of land price, which denotes the adjustment degree of benchmark land price controlled by government through the land supply plan. \( \alpha_2 \) is the adjustment parameter of housing price, the dynamic model of land price and housing price can be established as follows:
\[
\begin{align*}
p_1(t) &= p_1(t - 1) + \alpha_1 Z(p_1(t - 1)), \\
p_2(t) &= p_2(t - 1) + \alpha_2 Z(p_2(t - 1)), 
\end{align*}
\] (2.7)

where \( \alpha_1 \) and \( \alpha_2 \) are positive parameters.

It is clear that the excess functions of land and housing with adjustment parameters are two-dimensional nonlinear map, which can be regarded as a discrete dynamic system.

3. Stability analysis

3.1. Bifurcation and chaos. Expansion formula of (2.7) is:
\[
\begin{align*}
p_1(t) &= p_1(t - 1) + \alpha_1 \left[ b_0 - e_0 - (e_1 + b_1) p_1(t - 1) + (b_2 - e_2) p_1^2(t - 1) \right], \\
p_2(t) &= p_2(t - 1) + \alpha_2 \left[ c_0 - d_0 - (d_1 + c_1) p_2(t - 1) + (c_2 - d_2) p_2^2(t - 1) + d_3 p_1(t - 1) \right], 
\end{align*}
\] (3.1)

Let
\[
\begin{align*}
U &= \frac{1}{2} - \frac{1}{2} \cdot \frac{\alpha_1 (e_2 - b_2)}{1 + \alpha_1 \sqrt{(e_1 + b_1)^2 - 4(e_0 - b_0)(e_2 - b_2)}}, \\
x(t) &= u p_1(t) + U. 
\end{align*}
\] (3.2)
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The transform of the first equation in formula (3.1) is:

\[ x(t) = \lambda x(t - 1)(1 - x(t - 1)), \]  

(3.3)

where \( \lambda = 1 + \alpha_1 \sqrt{(e_1 + b_1)^2 - 4(e_2 - b_2)(e_0 - b_0)} \). The stability of \( x(t) \) varies along with variety of \( \lambda \) according to Logistic rule.

If \( \alpha_1 < 0 \), then \( \lambda < 1 \) implies that one fixed point exists in system (3.1), however, \( \alpha_1 < 0 \) is insignificant, so we do not give consideration.

If \( 0 \leq \alpha_1 < 2/\sqrt{(e_1 + b_1)^2 - 4(e_2 - b_2)(e_0 - b_0)} \), then \( 1 \leq \lambda < 3 \) implies that two fixed points exist in system (3.1) and period-doubling bifurcation appears.

If \( 2/\sqrt{(e_1 + b_1)^2 - 4(e_2 - b_2)(e_0 - b_0)} \leq \alpha_1 \leq \sqrt{6}/\sqrt{(e_1 + b_1)^2 - 4(e_2 - b_2)(e_0 - b_0)} \), then \( 3 \leq \lambda \leq 1 + \sqrt{6} \) implies that four fixed points exist in system (3.1) and period-doubling bifurcation appears.

As \( \lambda \) increases, the number of fixed points continues to grow until \( \lambda = 3.5699 \); when \( \alpha_1 = 2.5699/\sqrt{(e_1 + b_1)^2 - 4(e_2 - b_2)(e_0 - b_0)} \), the value of \( x(t) \) is unequal to any point that appeared before, system enters chaos from period doubling bifurcation.

The same argument holds for the second equation in formula (3.1). Let \( \alpha_1 = 0 \):

When \( 0 < \alpha_2 < 2/\sqrt{(d_1 + c_1)^2 - 4(d_0 - c_0)(d_2 - c_2)} \), bifurcation occurs.

When \( 2/\sqrt{(d_1 + c_1)^2 - 4(d_0 - c_0)(d_2 - c_2)} \leq \alpha_2 \leq \sqrt{6}/\sqrt{(d_1 + c_1)^2 - 4(d_0 - c_0)(d_2 - c_2)} \), period doubling bifurcation occurs.

When \( \alpha_2 = 2.5699/\sqrt{(d_1 + c_1)^2 - 4(d_0 - c_0)(d_2 - c_2)} \), system enters chaotic state from period doubling bifurcation.

3.2 Stability analysis. Now we discuss the stability of fixed points of the discrete dynamic system (3.1) through analyzing the eigenvalues of asymptotic linear equation of formula (3.1).

Four fixed points of difference function (3.1) are obtained:

\[
E : \begin{cases}
  p_1 = \frac{b_1 + e_1 \pm \sqrt{(b_1 + e_1)^2 - 4(e_2 - b_2)(e_0 - b_0)}}{2(b_2 - e_2)}, \\
  p_2 = \frac{d_1 + c_1 \pm \sqrt{(d_1 + c_1)^2 - 4(c_2 - d_2)(c_0 - d_0 + d_3 p_1)}}{2(c_2 - d_2)},
\end{cases}
\]

(3.4)

provided that:

\[
(b_1 + e_1)^2 - 4(e_2 - b_2)(e_0 - b_0) \geq 0,
\]

(3.5)

\[
(d_1 + c_1)^2 - 4(c_2 - d_2)(c_0 - d_0 + d_3 p_1) \geq 0.
\]
Lemma 3.1. The equilibrium

\[
E_1 \left( b_1 + e_1 + \sqrt{ \frac{(b_1 + e_1)^2 - 4(e_2 - b_2)(e_0 - b_0)}{2(b_2 - e_2)} }, \frac{d_1 + c_1 + \sqrt{ (d_1 + c_1)^2 - 4(c_2 - d_2)(c_0 - d_0 + d_3 p_1)} }{2(c_2 - d_2)} \right)
\]

(3.6)

is an unstable equilibrium point.

Proof. In order to prove this result, we find the eigenvalues of the Jacobian matrix \( J \). In fact at \( E_1 \), the Jacobian matrix becomes a triangular matrix:

\[
J(E_1) = \begin{bmatrix}
1 + \alpha_1 \left[ (b_1 + e_1)^2 - 4(e_2 - b_2)(e_0 - b_0) \right]^{1/2} & 0 \\
\alpha_2 d_3 & 1 + \alpha_2 \left[ (d_1 + c_1)^2 - 4(c_2 - d_2)(c_0 - d_0 + d_3 p_1) \right]^{1/2}
\end{bmatrix}
\]

(3.7)

whose eigenvalues are given by the diagonal entries. They are:

\[
\lambda_1 = 1 + \alpha_1 \left[ (b_1 + e_1)^2 - 4(e_2 - b_2)(e_0 - b_0) \right]^{1/2},
\]

\[
\lambda_2 = 1 + \alpha_2 \left[ (d_1 + c_1)^2 - 4(c_2 - d_2)(c_0 - d_0 + d_3 p_1) \right]^{1/2}.
\]

(3.8)

It is clear that when condition (3.5) holds, then \( |\lambda_1| > 1 \) and \( |\lambda_2| > 1 \). Then \( E_1 \) is an unstable equilibrium point of the system (3.1). This completes the proof of the proposition.

The stability of other points can also be judged by the above method. \( \square \)

3.3. The stable region of equilibrium point. In this subsection, we analyze the asymptotic stability of the equilibrium point for the two-dimensional map (3.1). We determine the region of stability in the plane of the parameters \( (\alpha_1, \alpha_2) \). The Jacobian matrix at \( E^*(p_1^*(t), p_2^*(t)) \) takes the form

\[
J = \begin{bmatrix}
1 - \alpha_1(e_1 + b_1) + 2\alpha_1(b_2 - e_2) p_1^*(t) & 0 \\
\alpha_2 d_3 & 1 - \alpha_2(d_1 + c_1) + 2\alpha_2(c_2 - d_2) p_2^*(t)
\end{bmatrix}.
\]

(3.9)

The characteristic equation of the matrix (3.9) has the form

\[
F(\lambda) = \lambda^2 - \text{Tr}\lambda + \text{Det} = 0,
\]

(3.10)

where “\( \text{Tr} \)” is the trace and “\( \text{Det} \)” is the determinant of the Jacobian matrix (3.9) which are given by

\[
\text{Tr} = 2 - \alpha_1(e_1 + b_1) + 2\alpha_1(b_2 - e_2) p_1^*(t) - \alpha_2(d_1 + c_1) + 2\alpha_2(c_2 - d_2) p_2^*(t),
\]

\[
\text{Det} = 1 - \alpha_1(e_1 + b_1) + 2\alpha_1(b_2 - e_2) p_1^*(t) - \alpha_2(d_1 + c_1) + 2\alpha_2(c_2 - d_2) p_2^*(t) + [\alpha_1(e_1 + b_1) - 2\alpha_1(b_2 - e_2) p_1^*(t)][\alpha_2(d_1 + c_1) - 2\alpha_2(c_2 - d_2) p_2^*(t)].
\]

(3.11)
we deduce that the eigenvalues of equilibrium are real. The local stability of equilibrium point is given by Jury’s conditions [14, 15] which are as follows.

(a) $1 - \text{Tr} + \text{Det} > 0$.

**Lemma 3.2.** The condition (a) is always satisfied.

*Proof.* Because $1 - \text{Tr} + \text{Det} = [\alpha_1(e_1 + b_1) - 2\alpha_1(b_2 - e_2)p_1^*(t)][\alpha_2(d_1 + c_1) - 2\alpha_2(c_2 - d_2)p_2^*(t)]$, moreover, the last two conditions in (2.6) are hold, $\alpha_1$ and $\alpha_2$ are positive parameters, so the sign of “$1 - \text{Tr} + \text{Det}$” is positive and the lemma is proven.

(b) $1 + \text{Tr} + \text{Det} > 0$,

$$1 + \text{Tr} + \text{Det} = 4 - 2\alpha_1(e_1 + b_1) + 4\alpha_1(b_2 - e_2)p_1^*(t) - 2\alpha_2(d_1 + c_1) + 4\alpha_2(c_2 - d_2)p_2^*(t) + [\alpha_1(e_1 + b_1) - 2\alpha_1(b_2 - e_2)p_1^*(t)][\alpha_2(d_1 + c_1) - 2\alpha_2(c_2 - d_2)p_2^*(t)].$$

(c) $\text{Det} - 1 < 0$,

$$\text{Det} - 1 = -\alpha_1(e_1 + b_1) + 2\alpha_1(b_2 - e_2)p_1^*(t) - \alpha_2(d_1 + c_1) + 2\alpha_2(c_2 - d_2)p_2^*(t) + [\alpha_1(e_1 + b_1) - 2\alpha_1(b_2 - e_2)p_1^*(t)][\alpha_2(d_1 + c_1) - 2\alpha_2(c_2 - d_2)p_2^*(t)].$$

The conditions (b) and (c) define a bounded region of stability in the parameters space $(\alpha_1, \alpha_2)$. Then the second and third conditions are the conditions for the local stability of equilibrium point which becomes

$$1 + \text{Tr} + \text{Det} = 4 - 2\alpha_1(e_1 + b_1) + 4\alpha_1(b_2 - e_2)p_1^*(t) - 2\alpha_2(d_1 + c_1) + 4\alpha_2(c_2 - d_2)p_2^*(t) + [\alpha_1(e_1 + b_1) - 2\alpha_1(b_2 - e_2)p_1^*(t)][\alpha_2(d_1 + c_1) - 2\alpha_2(c_2 - d_2)p_2^*(t)] > 0,$$

$$\text{Det} - 1 = -\alpha_1(e_1 + b_1) + 2\alpha_1(b_2 - e_2)p_1^*(t) - \alpha_2(d_1 + c_1) + 2\alpha_2(c_2 - d_2)p_2^*(t) + [\alpha_1(e_1 + b_1) - 2\alpha_1(b_2 - e_2)p_1^*(t)][\alpha_2(d_1 + c_1) - 2\alpha_2(c_2 - d_2)p_2^*(t)] < 0.$$

The stability region is bounded by the portion of hyperbola with positive values of $\alpha_1$ and $\alpha_2$, whose equations are given by the vanishing of the left-hand sides $1 + \text{Tr} + \text{Det} = 0$ and $\text{Det} - 1 = 0$. For the values of $(\alpha_1, \alpha_2)$ inside the stability region (see Figure 3.1), the equilibrium point is stable node and loses its stability through a period-doubling bifurcation. The bifurcation curve intersects the axes $\alpha_1$ and $\alpha_2$, respectively, whose coordinates are given by

$$A\left(0, \frac{2}{d_1 + c_1 - 2(c_2 - d_2)p_2^*(t)}\right), \quad B\left(\frac{2}{e_1 + b_1 - 2(b_2 - e_2)p_1^*(t)}, 0\right).$$

□
4. Numerical simulations

In order to study the complex dynamics of system (3.1), it is convenient to take the parameters values as follows:

\[
\begin{align*}
    b_0 &= 1.2, & b_1 &= 2, & b_2 &= 1.6, & c_0 &= 4, & c_1 &= 1.6, & c_2 &= 0.04, & d_0 &= 0, \\
    d_1 &= 3, & d_2 &= 0.02, & d_3 &= 0.4, & e_0 &= 0.5, & e_1 &= 0.3, & e_2 &= 0.2.
\end{align*}
\]

Figure 3.1 shows the region of stability of Nash equilibrium. Equation (3.15) defines the region of stability in the plane of \((\alpha_1, \alpha_2)\). Figure 4.1 shows the map of \(f_{\alpha_1, \alpha_2}\). X-coordinate is \(p_1\) and y-coordinate is \(f_{\alpha_1, \alpha_2}(p_1)\). Dynamics of land price in the cobweb model is given by system \(p_1(t) = f_{\alpha_1, \alpha_2}(p_1(t - 1))\) with two model parameters. A graphical analysis in Figure 4.1 shows that the map \(f_{\alpha_1, \alpha_2}\) is nonmonotonic with one critical
point, where the graph has a (local) minimum, and that initial state $p_1(0) = 1$ does not converge to a low periodic orbit. Since the graphical analysis in this case does not converge, it suggests that the dynamical behavior is chaotic.

Figures 4.2 and 4.3 show the bifurcation diagrams with respect to the parameter $\alpha_1$ and for $\alpha_2 = 0.2$ and 0.4. In both figures, the Nash equilibrium $E^* = (0.4, 0.9)$ is locally stable for small values of the parameter $\alpha_1$. If $\alpha_1$ increases, the Nash equilibrium point
becomes unstable, and one observes complex dynamic behavior occurs such as cycles of higher order and chaos. Also the maximal Lyapunov exponent is plotted in Figures 4.2 and 4.3.

Figures 4.4, 4.5, 4.6, and 4.7 show the graph of strange attractors for the different values of $\alpha_2$. The parameter $\alpha_2$ takes the values 0.07, 0.1, 0.2, and 0.3, which exhibit fractal structure in both cases.

We compute the difference of two orbits with initial points $[p_1(0), p_2(0)]$ and $[p_1(0) + 0.0001, p_2(0)]$, as well as $[p_1(0), p_2(0)]$ and $[p_1(0), p_2(0) + 0.0001]$, to demonstrate the sensitivity to initial conditions of the system (3.1). The parameters take the values $(\alpha_1, \alpha_2) = (2.3, 0.6)$ and $[p_1(0), p_2(0)] = (1, 2)$. The results are shown in Figures 4.8 and 4.9, where $\Delta p_1(t) = p_1(t) - p_1'(t)$ and $p_1'(t)$ is the value of land price at time period $t$ with initial
value of $p_1(0) + 0.0001; \Delta p_2(t) = p_2(t) - p_2'(t)$, and $p_2'(t)$ is the value of housing price at time period $t$ with initial value of $p_2(0) + 0.0001$. In both figures, initial condition of one coordinate differs by 0.0001, the other coordinate keeps equal. At the beginning, the difference is indistinguishable but after a number of iterations the difference between them builds up rapidly. From Figures 4.8 and 4.9, we show that the time series of the system (3.1) is sensitive dependence on initial conditions, that is, complex dynamics behaviors occur in this model.
5. Chaos control

Delay feedback control (DFC) method was brought forward by Pyragas [16]. The method allows a noninvasive stabilization of unstable periodic orbits (UPOs) of dynamical systems [17]. It feeds back part of system output signals as exterior input to the system after a time delay. $u(\bullet)$ is control signal gained by self-feedback coupling between output and input signals in chaotic system. $x(t) = f(x(t-1)) + u(t)$ is the form of DFC, where $u(t) = k(x(t) - x(t-\tau))$, $t > \tau$, $\tau$ is time delay, $k$ is controlling factor. Though delay feedback control is only carried out on one variable, it enables other variables in the system to achieve stability simultaneously. Our goal is to control the system in such way. The
system with controlling factor is shown as follows:

\[
\begin{align*}
    p_1(t) &= p_1(t - 1) + \alpha_1 Z(p_1(t - 1)) - k(p_1(t) - p_1(t - \tau)), \\
    p_2(t) &= p_2(t - 1) + \alpha_2 Z(p_2(t - 1)),
\end{align*}
\]

\[t = 0, 1, 2, \ldots\] (5.1)

From Figure 3.1, we know that chaos exists in system (3.1) when \(\alpha_2 = 0.4, \alpha_1 = 2.3\). Choosing \(\tau = 1\), first inspect the relation of \(k\) and system stability. The Jacobian matrix of
system (5.1) is
\[
J = \begin{bmatrix}
(1 - \alpha_1(e_1 + b_1) + k + 2\alpha_1(b_2 - e_2)p_1(t))/(1 + k) & 0 \\
\alpha_2d_1 & 1 - \alpha_2(d_1 + c_1) + 2\alpha_2(c_2 - d_2)p_2(t)
\end{bmatrix}.
\]

Substituting equilibrium point (0.4, 0.9) into (5.2), we obtain eigenvalues \(\lambda_1 = -0.83\), \(\lambda_2 = (k - 1.7)/(1 + k)\). So when \(k > 0.35\), absolute values of both eigenvalues are less than 1, which means that the system is stable.

As shown in Figure 4.10 land price is controlled from chaotic state to stable state when \(k\) is greater than 0.35, so we select \(k = 0.4\). Housing price and land price are also controlled to equilibrium point (0.4, 0.9) as shown in Figure 4.11.

6. Conclusion

A nonlinear model for real estate market has been presented based on the cobweb theory. It is a simple dynamic model with nonlinear demand and supply function. From numerical simulations, we deduce that the land supply system has the remarkable influence on real estate market. Therefore, policy makers who intervene in one market should recognize that what they do may also influence other relative markets. We showed that the fast adjustment cause a market structure to behave chaotically. Therefore, the dynamics of market is changed when players apply different adjustment speed. Attempts are also made to stabilize the chaotic system with the delay feedback method. Combining with this method, the land price and housing price evolve from chaotic to stable.

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Thinking about nonlinearity in engineering areas, up to the 70s, was focused on intentionally built nonlinear parts in order to improve the operational characteristics of a device or system. Keying, saturation, hysteretic phenomena, and dead zones were added to existing devices increasing their behavior diversity and precision. In this context, an intrinsic nonlinearity was treated just as a linear approximation, around equilibrium points.

Inspired on the rediscovering of the richness of nonlinear and chaotic phenomena, engineers started using analytical tools from “Qualitative Theory of Differential Equations,” allowing more precise analysis and synthesis, in order to produce new vital products and services. Bifurcation theory, dynamical systems and chaos started to be part of the mandatory set of tools for design engineers.

This proposed special edition of the Mathematical Problems in Engineering aims to provide a picture of the importance of the bifurcation theory, relating it with nonlinear and chaotic dynamics for natural and engineered systems. Ideas of how this dynamics can be captured through precisely tailored real and numerical experiments and understanding by the combination of specific tools that associate dynamical system theory and geometric tools in a very clever, sophisticated, and at the same time simple and unique analytical environment are the subject of this issue, allowing new methods to design high-precision devices and equipment.

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