ABSTRACT. When q is an interpolating Blaschke product, we find necessary and sufficient conditions for a subalgebra B of $H^\infty[q]$ to be a maximal subalgebra in terms of the nonanalytic points of the noninvertible interpolating Blaschke products in B. If the set $M(B) \cap Z(q)$ is not open in $Z(q)$, we also find a condition that guarantees the existence of a factor $q_0$ of q in $H^\infty$ such that B is maximal in $H^\infty[q]$. We also give conditions that show when two arbitrary Douglas algebras A and B, with $A \subseteq B$ have property that A is maximal in B.

KEY WORDS AND PHRASES. Maximal subalgebra, Douglas algebra, interpolating sequence, sparse sequence, Blaschke product, inner functions, open and closed subset, nonanalytic points, support set, Q-C level sets.


1. INTRODUCTION.

Let $D$ be the open unit disk in the complex plane and $T$ be its boundary. Let $L^\infty$ be the space of essentially measurable functions on $T$ with respect to the Lebesgue measure. By $H^\infty$ we mean the family of all bounded analytic functions in $D$. Via identification with boundary functions, $H^\infty$ can be considered as a uniformly closed subalgebra of $L^\infty$. A uniformly closed subalgebra B between $H^\infty$ and $L^\infty$ is called a Douglas algebra. If we let $C$ be the family of continuous functions on $T$, then it is well known that $H^\infty + C$ is the smallest Douglas algebra containing $H^\infty$ properly. For any Douglas algebra B, we denote by $M(B)$ the space of nonzero multiplicative linear functionals on B, that is, the set of all maximal ideals in B. An algebra $B_0$ is said to be a maximal subalgebra of B, if $B_1$ is another algebra with the property that $B_0 \subseteq B_1 \subseteq B$, then either $B_1 = B_0$ or $B_1 = B$.

An interpolating sequence $\{z_n\}_{n=1}^\infty$ is a sequence in $D$ with the property that for any bounded sequence of complex numbers $\{\lambda_n\}_{n=1}^\infty$, there exists $f$ in $H^\infty$ such that $f(z_n) = \lambda_n$ for all $n$. A well-known condition states that a sequence $\{z_n\}_{n=1}^\infty$ is interpolating if and only if

$$\inf_{n \neq m} \left| \frac{z_m - z_n}{1 - \overline{z_n} z_m} \right| = \delta > 0.$$  \hspace{1cm} (1.1)
A Blaschke product

\[ q(z) = \prod_{n=1}^{\infty} \frac{|z_n|}{z_n} \left( \frac{z - z_n}{1 - \overline{z_n} z} \right) \]

is called an interpolating Blaschke product if its zero set \( \{z_n\}_{n=1}^{\infty} \) is an interpolating sequence (\( |z_n|/z_n \equiv 1 \) is understood whenever \( z_n = 0 \)). A sequence \( \{z_n\}_{n=1}^{\infty} \) is said to be sparse if it is an interpolating sequence and

\[ \lim_{n \to \infty} \prod_{n \neq m} \frac{|z_m - z_n|}{1 - \overline{z_n} z_m} = 1. \]

For a function \( q \) in \( H^\infty + C \), we let \( Z(q) = \{m \in M(H^\infty + C) : q(m) = 0\} \) be the zero set of \( \phi \) in \( M(H^\infty + C) \). An inner function is a function in \( H^\infty \) of modulus 1 almost everywhere on \( T \). We denote by \( H^\infty \) the Douglas algebra generated by \( H^\infty \) and the complex conjugate of the inner function \( b \).

We put \( X = M(L^\infty) \). Then \( X \) is the Shilov boundary for every Douglas algebra. For a point in \( M(H^\infty) \), we denote by \( \mu_x \) the representing measure on \( X \) for \( x \) and by \( \text{supp} \mu_x \) the support set for \( \mu_x \).

For a function \( q \) in \( L^\infty \) (in particular if \( q \) is an interpolating Blaschke product), we put \( N(q) \) the closure of the union set of \( \text{supp} \mu_x \) such that \( x \in M(H^\infty + C) \) and \( \overline{q}\mu_X \supseteq H^\infty \text{supp} \mu_x \). Roughly speaking, \( N(q) \) is the set of nonanalytic points of \( q \). Set \( QC = H^\infty + C \cap H^\infty + C \) and for \( x_0 \) in \( X \), let \( Q_{x_0} = \{f(x) = f(x_0) \text{ for } f \in QC\} \).

\( x_0 \) is called the QC-level set for \( x_0 \) [9].

**THEOREM 1.** If \( q \) is an inner function that is not a finite Blaschke product, then

\[ N(q) = \bigcup \{Q_x ; x \in Z(q)\}. \]

In particular, the right side of 1.4 is a closed set. Now assume that \( q \) is an interpolating Blaschke product, and let \( B \) be a Douglas algebra contained in \( H^\infty[q] \). We will always assume that \( M(B) \cap Z(q) \) is not an open set in \( Z(q) \), for Izuchi has shown [6] that if \( B \) is a maximal subalgebra of \( H^\infty[q] \), then \( M(B) \cap Z(q) \) is not open in \( Z(q) \). We will give answers to the following two questions. When is \( B \) a maximal subalgebra of \( H^\infty[q] \) or when is there a factor \( q_0 \) of \( q \) in \( H^\infty[q] \) such that \( B \) is maximal in \( H^\infty[q] \)? These
answers will be in terms of the nonanalytic points of \( q \) and the invertible inner functions of \( H^\infty[q] \) that are not invertible in \( B \).

For a Douglas algebra \( B \), we denote by \( N(B) \) the closure of \( \{\text{supp} \mu_X ; x \in M(H^\infty + C)/M(B)\} \).

In particular \( N(H^\infty[q]) = N(q) \). In general if \( A \) and \( B \) are Douglas algebras such that
A \subseteq B$, we put $N_A^a(B) =$ the closure of $\cup \{\text{supp } \mu_x : x \in M(A)/M(B)\}$ and for any inner function $b$, $N_A^b(b) =$ the closure of $\cup \{\text{supp } \mu_x : x \in M(A), |b(x)| < 1\}$.

It is shown in [7, Corollary 2.5] that if $B \subseteq H^\infty[\overline{q}]$, then $N(B) \subseteq N(q)$, and it is not hard to show that $N(q)/N(B) \supseteq N_B(q)$ (in a sense the set $N_B(q)$ is generated by the nonanalytic points $M(B)/M(H^\infty[\overline{q}]) \subseteq M(H^\infty + C)/M(H^\infty[\overline{q}])$).

2. OUR MAIN RESULT.

We'll need the following lemma. It shows how small $M(B)/M(H^\infty[\overline{q}])$ must be if $B$ is to be a maximal subalgebra of $H^\infty[\overline{q}]$. Let $\Omega =$ \{b : b is an interpolating Blaschke product $\}$ with $b \subseteq H^\infty[\overline{q}]$, and $\Omega(B) =$ \{b_0 \in \Omega : b_0 \notin B\}.

**LEMMA 1.** Let $q$ be an interpolating Blaschke product and $B$ be a Douglas algebra contained in $H^\infty[q]$. Suppose for all $b_0 \in \Omega(B)$, we have that $N_B(q) \subseteq N_B(b_0)$. Then $B$ is a maximal subalgebra of $H^\infty[q]$.

**PROOF.** It suffices to show that if $b \in \Omega(B)$, then $B[b] \subseteq H^\infty[\overline{q}]$. Hence the only Douglas algebra between $B$ and $H^\infty[\overline{q}]$ that contains $B$ properly is $H^\infty[\overline{q}]$. It is clear that $M(H^\infty[\overline{q}]) \subseteq M(B[b])$. We show that $M(B[b]) \subseteq M(H^\infty[\overline{q}])$. Now $M(B[b]) =$ \{m \in M(B) : |b(m)| = 1\}.

It suffices to show that if $m \notin M(H^\infty[\overline{q}])$, then $m \notin M(B[b])$. Let $m \in M(B)$ such that $m \notin M(H^\infty[\overline{q}])$. Then $m \mid \text{supp } \mu_{\infty} \in H^\infty[\overline{q}]$ and since $N_B(q) \subseteq N_B(b)$, we have that $b \mid \text{supp } \mu_{\infty} \in H^\infty[\overline{q}]$. Thus $|b(m)| < 1$ and we get $m \notin M(B[b])$. This shows that $M(B[b]) \subseteq M(H^\infty[\overline{q}])$, and $B$ is maximal in $H^\infty[q]$.

Using Theorem 1 above, it is not hard to show directly that $N(B[b]) = N(q)$. However, by Proposition 4.1 of [7], this condition is not sufficient.

We let $E = N_B(q)$. This can be a very complicated set. For example, it can contain $\mu_x$ where $x$ belongs to a trivial Gleason part or a Gleason part where $|q| < 1$, but yet $q \neq 0$ on this part [see 3]. So for $B$ to be maximal in $H^\infty[q]$, $E$ must be as simple as possible. To see how simple, we set $\Lambda(B) =$ \{b \in \Omega(B) : b \subseteq H^\infty[\overline{b}]\} and $\Lambda^*(B) =$ \{a \in \Lambda(B) : a \notin \Lambda(B)\}. Now let $E^* = \cap N(b)$, $E^{**} = \cap N(b_0)$, $E^* = E \cap E$ and $E^{**} = E^{**} \cap E$. Note that if $E^{**} = \emptyset$, then there are interpolating Blaschke products $a_0$ and $a_1$ in $\Lambda^*(B)$ such that $N_B(q) \cap N(a_0) \cap N(a_1) = \emptyset$. Thus we get $B \subseteq B[a_0] \subseteq H^\infty[q]$. To see this, just note that both $N_B(q) \cap N(a_0) \neq \emptyset$ and $N_B(q) \cap N(a_1) \neq \emptyset$ since $a_0$ and $a_1$ belong to $\Lambda^*(B)$. Since their intersection is empty, there is an $x_1 \in M(B)$ such that $a_0 \mid \text{supp } \mu_{x_1} \in H^\infty[\overline{q}]$. Thus $N_B(a_0)(q) < N(q)$, which implies that $B[a_0] \subseteq H^\infty[q]$. Obviously, $B \subseteq B[a_0]$, so $B$ cannot be maximal in $H^\infty[q]$ unless $E^{**} \neq \emptyset$. We now state.

**PROPOSITION 1.** Let $B$ be a Douglas algebra properly contained in $H^\infty[q]$, and suppose $E^{**} \neq \emptyset$. Then the following statements are equivalent:

(i) $N(B) = N(q)$;

(ii) $B$ is a maximal subalgebra of $H^\infty[q]$;

(iii) $E^{**} = E^* = E$;

(iv) $E^* = N_B(q)$. 


PROOF. We prove the following: (i) + (ii) + (iii) + (iv) + (ii) + (i).

Suppose (i) holds. We will show that $N_B(q) \subseteq N_B(b)$ for all $b \in \Omega(B)$. Using Lemma 1, this will prove that $B$ is a maximal subalgebra of $H^\infty[q]$. Let $b \in \Omega(B)$ and consider the Douglas algebra $B[b]$. We have $B \subseteq B[b] \subseteq H^\infty[q]$, hence $N(B) = N(B[b]) = N(q)$. Now $N(q) = N(B) \cup N_B(q)$, so by the above equality we have that $N_B(q) \subseteq N(B[b])$. Thus, if $x \in M(B)$ such that $\supp \frac{\mu}{x} H^\infty$ implies that $\supp \frac{\mu}{x} \subseteq N(B[b])$. Thus $\supp \frac{\mu}{x} \subseteq N_B(q)$. We have (i) + (ii).

Next suppose that (ii) holds. It is clear that $E_0 \subseteq E_0^{**} = E$. We must show that $E_0^{**} = E$. First, suppose that $E_0^{**}$ is empty. Suppose not. Then there is an $x \in M(B)$ and a $b_0 \in \Lambda(B)$ such that $\supp \frac{\mu}{x} H^\infty$, and $\supp \frac{\mu}{x} \subseteq E_0^{**}$. It is clear by Theorem 1 that $\supp \frac{\mu}{x} \cap N(B_0) = \emptyset$. Consider the algebra $B[b_0]$. Since $b_0 \in \Lambda(B)$, $E \subseteq B[b_0]$. Since $\supp \frac{\mu}{x} \subseteq N(q)$ and $\supp \frac{\mu}{x} \not\subseteq N(B_0)$, we have that $|b_0(x)| = 1$, so we have $\supp \frac{\mu}{x} \subseteq N(q)/N_B[0](q)$. This implies that $B[b_0] \subseteq H^\infty[q]$, which is a contradiction.

Now we show that $E_0 = E$. Again suppose not. Hence there is a $y \in M(B)$ such that $\supp \frac{\mu}{y} \subseteq E$, but $\supp \frac{\mu}{y} \not\subseteq E_0$. There is a $b \in \Lambda(B)$ such that $\supp \frac{\mu}{y} \subseteq N(B[b])$. Again this implies that $\supp \frac{\mu}{y} H^\infty$, so we have that $\supp \frac{\mu}{y} \not\subseteq E_0$. Thus we have that $B \not\subseteq B[b]$ (since $b \in \Lambda(B)$) and $B[b] \subseteq H^\infty[q]$ (since $\supp \frac{\mu}{y} \subseteq N(q)/N_B[0](q)$), which is a contradiction.

So we get $E_0 = E$. This shows that (ii) + (iii).

It is trivial that if (iii) holds, $E_0^{**} = E_0$.

If (iv) holds and $b$ is any interpolating Blaschke product in $\Omega(B)$, then by (iv) $N_B(q) \subseteq N_B(b)$ so by Lemma 1, $B$ is a maximal subalgebra of $H^\infty[q]$.

Finally, suppose (ii) holds. We are going to show that $N(B) = N(q)$. Suppose not. Then $N(B) \not\subseteq N(q)$. By Theorem 1 there is a $Q$-C level set $Q$ with $N(B) \cap Q = \emptyset$. Put $B_0 = [H^\infty, I]$; $I$ is an interpolating Blaschke product with $I \in H^\infty[q]$ and $I \in I_0 \subseteq H^\infty[q]$. By Proposition 4.1 of [7], we have $B_0 \subseteq H^\infty[q]$ and $N(B_0) = N(q)$. Since $N(B) \cap Q = \emptyset$, we also have $B \subseteq B_0$ (because $N(B) \subseteq N(B_0)$). This implies that $B$ is not a maximal subalgebra of $H^\infty[q]$, which is a contradiction. Thus $N(B) = N(q)$.

Now suppose we have that $E_0^{**} = E_0 \subseteq E$ ($E_0^{**} = \emptyset$ is possible).

When is there a factor $q_0$ of $q$ in $H^\infty$ such that $B$ is a maximal subalgebra of $H^\infty[q_0]$? To answer this question, let $\Omega_0 = \{q_0 : q_0 \in H^\infty[q_0]\}$, and $\Omega_0(B) = \{q_0 \in \Omega_0 : B \subseteq H^\infty[q_0]\}$.

Set $F = \bigcap_{q_0 \in \Omega_0(B)} N(q_0)$. Suppose $F = N(q_0)$ for some factor $q_0$ of $q$ in $H^\infty$. Then $B \subseteq H^\infty[q_0]$. So $q_0$ is our possible candidate. Next, let $\Omega_0 = \{c : c$ is an interpolating Blaschke product with $c \in H^\infty[q_0]\}$,
\( \Omega_q(B) = \Omega \cap \Omega(B), \Lambda_q(B) = \Omega \cap \Lambda(B), \Lambda^*(B) = \Omega \cap \Lambda^*(B), \)

\[ F_0 = E \cap N(q_0), \quad F^* = F_0 \cap F, \quad F^{**} = \bigcap_{c \in \Omega_q(B)} N(c), \quad F_0^* = F^* \cap F_0, \quad \text{and finally} \]

\[ F_0^{**} = F^{**} \cap F_0. \]

We have the following.

**Corollary 1.** Let \( q_0 \) be a factor of \( q \) in \( \mathcal{H} \) such that \( F = N(q_0) \) and assume \( F_0^* \neq \emptyset \).

If any of the following conditions hold:

(i) \( F_0 = F = F_0^* \)

(ii) \( F_0^{**} = N_B(H_0), \) where \( H_0 = \bigcap_{q_0 \in \Omega(B)} \mathcal{H} \).

Then \( B \) is a maximal subalgebra of \( H_0 = \mathcal{H}[q_0] \) where \( q_0 \in \Omega(B) \).

The fact that \( F = N(q_0) \) for some \( q_0 \in \Omega(B) \) implies that \( H_0 = \mathcal{H}[q_0] \) and our corollary follows from Proposition 1.

We now consider this question for the general Douglas algebras. Let \( A \) and \( B \) be Douglas algebras such that \( A \subseteq B \) and there is an inner function \( q \) with \( B \subseteq A[q] \).

When this occurs we say that \( A \) is near \( B \). It is well known that if \( B = \mathcal{L}^\infty \) and \( A \) is any Douglas algebra properly contained in \( B \), then \( A \) is not near \( B \); that is, \( B \not\subseteq A[q] \) for any inner function \( q \). In fact \( \mathcal{L}^\infty \) is not countably generated over any Douglas algebra \( A \) [10]. So by the results of C. Sundberg [10] any Douglas algebra \( B \) which is countably generated over \( A \) is also near it.

The following result comes from [2, Lemma 5] and gives equivalent conditions for two Douglas algebras to be near each other [see 11, Theorem 1 for a similar result].

**Theorem 2.** Let \( A \) and \( B \) be Douglas algebras with \( \mathcal{H} \subset A \subset B \) and \( q \) be an inner function. Then the following statements are equivalent.

(i) \( M(A) = Z_A(q) \cup M(B) \)

(ii) \( \emptyset \not\subseteq A \).

where \( Z_A(q) = Z(q) \cap M(A) \).

**Proof.** Assume (i) holds; we show that \( \emptyset \not\subseteq A \). Let \( b \) be any interpolating Blaschke product for which \( \overline{b} \) is in \( B \). If \( x \) is in \( Z_A(b) \), we show that \( x \) is also in \( Z_A(q) \). Now \( x \) is in \( M(A) \) and \( b(x) = 0 \) implies that \( x \) is not in \( M(B) \), since \( \overline{b} \) is in \( B \). So by (i) we have that \( x \) must be in \( Z_A(q) \). Thus \( Z_A(b) \subseteq Z_A(q) \), and by Theorem 1 of [4] we have \( \overline{b} \) is in \( A \). Now let \( f \) be any function in \( B \). By the Chang Marshall Theorem [1,8] there is a sequence of functions \( \{h_n\} \) in \( \mathcal{H} \) and a sequence of interpolating Blaschke products \( \{b_n\} \) with \( b_n \neq B \) for all \( n \), such that \( h_n b_n \to f \).

But \( h_n b_n \to f \) belongs to \( A \) since \( b_n \) is in \( A \) for all \( n \). This proves (ii).
Assume (ii) holds. Let \( x \) be in \( M(A) \) but not in \( M(B) \). Then there is an inner function \( b \) which is invertible in \( B \) such that \( |b(x)| < 1 \). For any positive integer \( n \), the function \( f_n = q^n b \) is in \( A \), so

\[
|g(x)| = |b(x)| |f_n(x)| \leq |b(x)|^n.
\]

Letting \( n \to \infty \) we get \( (x) = 0 \). This proves (i).

Set \( Z_B(q) = M(B) \cap Z_A(q) \) and \( Z_B^*(q) = Z_A(q)/Z_B(q) \); then \( M(A)/M(B) = \bigcup_{x \in Z_B^*(q)} P_x \), since \( M(A) = M(B) \cup Z_A(q) \).

As we have previously done, let \( \Omega(B,A) \) be the set of interpolating Blaschke products \( b \) such that \( b \in B \) but \( b \notin A \) and set \( \mathcal{W}^* = \bigcap_{b \in \Omega(B,A)} \mathcal{N}_A(b) \). We assume \( \mathcal{W}^* \neq \emptyset \).

Using Proposition 1, Theorem 2 and Lemma 1, we have the following result.

**PROPOSITION 2.** Let \( A \) and \( B \) be arbitrary Douglas algebras such that \( A \) is near \( B \). Then the following statements are equivalent:

(i) \( N_A(B) \subseteq N_A(\mathcal{W}^*) \) for all \( b \in \Omega(B,A) \);

(ii) \( A \) is a maximal subalgebra of \( B \);

(iii) \( \mathcal{W}^* = \mathcal{N}_A(B) \).

**PROOF.** Assume that (i) holds. Since \( A \) is near to \( B \), there is an inner function such that \( M(A) = M(B) \cup \{ \bigcup_{x \in Z_B^*(\phi)} P_x \} \). If we set \( \mathcal{A}^* = \bigcup_{x \in Z_B^*(\phi)} P_x \), then it is immediate that

\[
N_A(B) = \text{closure of } \{ \supp \mu_x : x \in \mathcal{A}^* \}.
\]

Let \( b \) be any element in \( \Omega(B,A) \). By (i) we have that \( N_A(B) \subseteq N_A(\mathcal{W}^*) \). As in proof of Lemma 1 we have that \( A[b] = B \). Thus is a maximal in \( B \).

Assume that (ii) holds, and let \( x \in \mathcal{A}^* \). Since \( A \) is near \( B \), we have that \( M(A) = M(B) \cup \mathcal{A}^* \). If \( b \in \Omega(B,A) \), then by our hypothesis \( A[b] = B \), which implies that if \( y \in M(A) \) and \( |b(y)| = 1 \), then \( y \notin M(B) \) (since \( M(A[b]) = \{ g \in M(A) : |b(g)| = 1 \} = M(B) \)). So, if \( \supp \mu_x \subseteq N_A(B) \), the \( \mathcal{L}_x \mathcal{H}^* \supp \mu_x \). Thus \( N_A(B) \subseteq N_A(\mathcal{W}^*) \) for all \( b \in \Omega(B,A) \).

This implies that \( N_A(B) \subseteq \mathcal{W}^* \).

To show what \( \mathcal{W}^* \subseteq N_A(B) \), let \( b \in \Omega(B,A) \). Hence \( b \notin B \); therefore we have

\[
N_A(\mathcal{W}^*) = \text{closure of } \{ \supp \mu_x : x \in M(A), |b(x)| < 1 \} = \text{closure of } \{ \supp \mu_x : x \in M(A)/M(B), |b(x)| < 1 \} \subseteq \text{closure of } \{ \supp \mu_x : x \in M(A)/M(B) \} = N_A(B).
\]
Since this is true for any $b \in \sigma(B,A)$, we have $N_A(B) \supseteq W_A^*$. Thus $W_A^* = N_A(B)$ if $A$ is maximal in $B$.

It is trivial that if (iii) holds, $N_A(B) \subseteq N_A(b)$ for all $b \in \sigma(B,A)$.

We are done.

In Proposition 4.1 of [7] Izuchi constructed a family of Douglas algebras $B$ contained in $H'[q]$ with the property that $N(B) = N(q)$. By Proposition 1, we have that this family is a family of maximal subalgebras of $H'[q]$.

Finally we close this paper with the following question that I have been unable to answer.

QUESTION 1. Recall that if $q$ is an interpolating Blaschke product, then $N(q) = N(B) \cup N_B(q)$ for any Douglas algebra with $B \subseteq H'[q]$. Does there exist a Douglas algebra $B_0 \subseteq H'[q]$ with $N_{B_0}(q) = N(q)$?

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</tr>
</thead>
<tbody>
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</tr>
<tr>
<td>First Round of Reviews</td>
<td>March 1, 2009</td>
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