FOURIER TRANSFORMS IN GENERALIZED FOCK SPACES

JOHN SCHMEELK

Department of Mathematical Sciences
Box 2014, Oliver Hall, 1015 W. Main Street
Virginia Commonwealth University
Richmond, VA 23284-2014

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ABSTRACT. A classical Fock space consists of functions of the form,

$$\phi \leftrightarrow (\phi_0, \phi_1, ..., \phi_q, ...),$$

where $\phi_0 \in C$ and $\phi_q \in L^2(\mathbb{R}^q), q > 1$. We will replace the $\phi_q$, $q > 1$ with

$q$-symmetric rapid descent test functions within tempered distribution theory. This

space is a natural generalization of a classical Fock space as seen by expanding

functionals having generalized Taylor series. The particular coefficients of such

series are multilinear functionals having tempered distributions as their domain.

The Fourier transform will be introduced into this setting. A theorem will be

proven relating the convergence of the transform to the parameter, $s$, which sweeps

out a scale of generalized Fock spaces.

KEY WORDS AND PHRASES. Generalized Fock Spaces, tempered distributions, Fourier

transforms, and rapid descent test functions.

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1. INTRODUCTION.

Rapid descent test functions, $S(\mathbb{R}^q)$, and their dual tempered distributions,

$S'(\mathbb{R}^q)$, are excellent spaces to do the analysis of the Fourier transform (Bogolubov

and Logunov [1], Constantinescu [2], Friedman [3], Gelfand and Shilov [4], and

Lighthill [5]). The classical Fourier transform analysis examines spaces having test

functions defined on a finite number of independent variables. By this we mean the

independent variables of a rapid descent test function, $\phi(t_1, ..., t_q)$, belonging to a $q$-

dimensional Euclidean space. This paper will indicate a method that will enjoy the

property that the number of independent variables becomes infinite, that is in some

sense the dimension, $q \to \infty$. The need for this analysis is essential in advanced

physics. An infinite number of particles are described by state vectors in a Fock

space. The classical results are developed in a Hilbert space. Traditionally the

Lebesque integrable functions, $L^p(\mathbb{R}^q)$, are implemented in the construction of a direct
sum of these spaces. However, when you want to describe a frequency of a particle the Fourier transform must be studied. This presents a significant problem since the kernel, $e^{-2\pi itw}$, does not belong to any $L^p(R^q)$ space. This kernel problem is solved in tempered distribution theory (Constantinescu [2], Gelfand and Shilov [4], Lighthill [5], and Zemanian [6]) but the infinite number of variables problem still remains. This paper will implement tempered distributions together with a holomorphic functional theory developed in Schmeelk [7-10] to solve the infinite number of variables problem.

We briefly recall in $S(R^q)$ and $S'(R^q)$ the Fourier transforms are respectively defined as

$$(F\phi)(\omega_1,...,\omega_q) \triangleq \int_{R^q} \exp(-2\pi i t_1 \omega_1 + ... + t_q \omega_q) \phi(t_1,...,t_q) dt_1 ... dt_q$$

and

$$\langle F\phi(\omega_1,...,\omega_q), \phi(t_1,...,t_q) \rangle = \langle F(\omega_1,...,\omega_q), F(\phi(t_1,...,t_q)) \rangle$$

for all $\phi(t_1,...,t_q) \in S(R^q)$ and all $\phi(\omega_1,...,\omega_q) \in S'(R^q)$. The advantages of $S(R^q)$ and $S'(R^q)$ are many but the fundamental result is that the Fourier transform exists a homeomorphism and has the appropriate derivative – multiplication property. This paper will not include a survey of the many Fourier transform properties which are contained in Constantinescu [2], Friedman [3], Zemanian [6], Bracewell [11], Gonzalez and Wintz [12], and Papoulis [13].

We will extend the Fourier transform into generalized Fock spaces. The principle result will be the existence of the transform in the scale of Frechet spaces

$$\Gamma^p,s^B = \bigcup_{s \geq 1} \Gamma^p,s^B$$

and its corresponding dual, $(\Gamma^p,s^B)'$. A comprehensive examination of these spaces are contained in Schmeelk [7-10]. We will only review these spaces in sections 2 and 3.

2. THE SPACE, $\Gamma^p,s^B$

For each $s \geq 1$, the space $\Gamma^p,s^B(p > 1, B = (B_i^q = 0, B_i > B_j, j > i)$, is called an infinite dimensional Fock space. Then $p$ and $B_i$, $i > 0$ are all real numbers. These spaces are topological spaces of complex valued functionals on $S'(R; \mathbb{C})$, the space of complex valued distributions. The functionals which are members of $\Gamma^p,s^B$ are all

$$C^\infty(S'(R); \mathbb{C})$$. The complex or real valued functionals enjoy similar properties. The real valued functionals which are members of $\Gamma^p,s^B$ are developed in Schmeelk [8].

We also require if $\phi \in \Gamma^p,s^B$, then

$$\phi(x) = \sum_{q=0}^{\infty} a_q x^q = \sum_{q=0}^{\infty} a_q [x,...,x]$$

(2.1)

where $a_0 \in \mathbb{C}$ and $a_q, q > 1$ are $q$-multilinear symmetric continuous functionals on the space, $S'(R)x...xS'(R)$, ($q$ copies, $q > 1$) to $\mathbb{C}$. We identify for each $\phi \in \Gamma^p,s^B$ the
associated Fock state vector,

\[
\phi \leftrightarrow \begin{bmatrix}
    a_0 \\
a_1 \\
    \vdots \\
a_q \\
    \vdots
\end{bmatrix}
\]  
(2.2)

We equip our infinite dimensional Fock vector space with the following increasing sequence of norms:

\[
||| \phi |||_{m} = \sup_{q} \left| \frac{||a_q||_{m}^{1/p}}{(sB)^{q}} \right|, \quad m = 0,1,\ldots
\]  
(2.3)

where

\[
||a_q||_{m} = \sup_{x \in S'(R)} \left| a_q x^q \right|, \quad x \in S'(R), \quad m = 0,1,\ldots
\]  
(2.4)

with

\[
||x||_{m} = \sup_{\phi \in S(R)} ||\phi||_{m}^{1/p}, \quad \phi \in S(R), \quad m = 0,1,\ldots
\]  
(2.5)

and

\[
||| \phi |||_{m} = \sup_{\alpha \in \mathbb{N}^q} M(t_1,\ldots,t_q) |D^\alpha \phi(t_1,\ldots,t_q)|
\]  
(2.6)

where

\[
M(t_1,\ldots,t_q) \triangleq [(1+(2\pi t_1)^2)\ldots(1+(2\pi t_q)^2)]^m
\]  
(2.7)

and

\[
D^\alpha = \frac{\partial^{\alpha_1+\cdots+\alpha_q}}{\partial t_1^{\alpha_1}\cdots\partial t_q^{\alpha_q}}
\]

The norms defined in expression (2.6) using the functions $M(t_1,\ldots,t_q)$ so defined generate a sequence of norms equivalent to the sequence of norms implementing the functions, $M'(t_1,\ldots,t_q) = [(1+|t_1|)^2\ldots(1+|t_q|)^2]^m$, [2,3].

It was proven in reference [10] that each real valued functional, $\phi \in P, sB$, has a kernel representation which remains valid for complex valued functionals. This representation is as follows,

\[
\phi \leftrightarrow \begin{bmatrix}
    \phi_0 \\
    \phi_1 \\
    \vdots \\
    \phi_q(t_1,\ldots,t_q)
\end{bmatrix}
\]  
(2.8)

where $\phi_0 = a_0$ and $\phi_q(t_1,\ldots,t_q)$ are symmetric complex valued rapid descent test functions, $S_+(\mathbb{R}^q)$ satisfying,

\[
||| \phi |||_{m} = \sup_{q} \left| \frac{||\phi_q||_{m}^{1/p}}{(sB)^{q}} \right|, \quad m = 0,1,\ldots
\]  
(2.9)

where

\[
||a_q||_{m} = \sup_{x \in S'(R)} \left| a_q x^q \right|, \quad x \in S'(R), \quad m = 0,1,\ldots
\]  
(2.4)

and

\[
||x||_{m} = \sup_{\phi \in S(R)} ||\phi||_{m}^{1/p}, \quad \phi \in S(R), \quad m = 0,1,\ldots
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\[
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||| \phi |||_{m} = \sup_{q} \left| \frac{||\phi_q||_{m}^{1/p}}{(sB)^{q}} \right|, \quad m = 0,1,\ldots
\]  
(2.9)

where
The representation for $\phi$ given in expression (2.8) enjoys the standard square summable property often times postulated for Fock functionals as seen by the following theorem.

**THEOREM 2.10.** Given a $\phi \in L^p, sB^q$, its kernel representation given in expression (2.8) satisfies

$$
\|\phi\|_2^2 + \sum_{q=1}^{\infty} \int |\phi_q(t_1,\ldots,t_q)|^2 dt_1,\ldots,dt_q < \infty.
$$

**PROOF.** Clearly the constant, $|\phi_0|^2$, does not contribute to the convergence problem of the result of the theorem. Also since $\phi \in L^p, sB^q$, then by the requirement given in expression (2.9) there must exist a sequence of positive constants, $(C_m)_{m=0}^\infty$, such that

$$
\sup_{q} \|\phi_q\|_m \leq \frac{C_{sB}^m q^q}{q!^{1/p}}
$$

for all $q$ and $m = 0, 1, \ldots$. We now consider a partial sum,

$$
\sum_{q=1}^{q_0} \int |\phi_q(t_1,\ldots,t_q)|^2 dt_1\ldots dt_q
$$

$$
\frac{\left[ \sum_{q=1}^{q_0} \frac{M_{2m}(t_1,\ldots,t_q)}{q!} \int |\phi_q(t_1,\ldots,t_q)|^2 dt_1\ldots dt_q \right]^{1/p} q!^{1/p}(sB_m)^q}{q!^{1/p}(sB_m)^q}
$$

$$
\cdot \left[ \sum_{q=1}^{q_0} \frac{1}{q!} \int \frac{M_{2m}(t_1,\ldots,t_q)}{q!} dt_1\ldots dt_q \right]^{1/p} q!^{1/p}(sB_m)^q
$$

$$
\cdot \left[ \sum_{q=1}^{q_0} \frac{1}{q!^{2/p}(sB_m)^q} \right] q!^{1/p}(sB_m)^q
$$

$$
= \sum_{q=1}^{q_0} \frac{\|\phi_q\|_m q!^{1/p}}{q!^{1/p}(sB_m)^q} \|\phi_q\|_m q!^{1/p}(sB_m)^q
$$

$$
\leq \sum_{q=1}^{q_0} \frac{\|\phi_q\|_m q!^{1/p}}{q!^{2/p}(sB_m)^q} q!^{1/p}(sB_m)^q
$$

Since expression (2.11) converges for any $q_0$, the result follows.
3. THE FOURIER TRANSFORM IN $r_p^{SB}$.

DEFINITION 3.1. The Fourier transform $\mathcal{F}$ on $\phi \in r_p^{SB}$ is defined as follows,

$$
\mathcal{F} : \begin{bmatrix}
\phi_0 \\
\phi_1 \\
\vdots \\
\phi_q \\
\vdots \\
\end{bmatrix}
\mapsto
\begin{bmatrix}
\phi_0 \\
\int \exp\left[-2\pi i t_1 w_1\right] \phi_1(t_1) dt_1 \\
\vdots \\
\int \exp\left[-2\pi i (t_1 w_1 + \ldots + t_q w_q)\right] \phi_q(t_1, \ldots, t_q) dt_1 \ldots dt_q \\
\vdots \\
\end{bmatrix}
$$

LEMMA 3.2. $\mathcal{F}(\phi)$ is well defined for every $\phi \in r_p^{SB}$ and moreover

$$
\phi_0 + \sum_{q=1}^{\infty} \int_{R_q} \exp \left[-2\pi i (t_1 w_1 + \ldots + t_q w_q)\right] \phi_q(t_1, \ldots, t_q) dt_1 \ldots dt_q < \infty.
$$

PROOF. $\phi \in r_p^{SB}$ implies $\phi \in r_p^{s'B}$ for some $s' > 1$.

We then have

$$
\phi_0 + \sum_{q=1}^{\infty} \int_{R_q} \exp \left[-2\pi i (t_1 w_1 + \ldots + t_q w_q)\right] \phi_q(t_1, \ldots, t_q) dt_1 \ldots dt_q < \infty.
$$

$$
< \phi_0 + \sum_{q=1}^{\infty} \int_{R_q} \frac{M_1(t_1, \ldots, t_q)}{M_1(t_1, \ldots, t_q)} \phi_q(t_1, \ldots, t_q) dt_1 \ldots dt_q
$$

$$
< \left\| \phi_0 \right\| + \sup_{(t_1, \ldots, t_q) \in R_q} \frac{M_1(t_1, \ldots, t_q)}{M_1(t_1, \ldots, t_q)} \left\| \phi(t_1, \ldots, t_q) \right\| \sum_{q=1}^{\infty} \int_{R_q} \frac{1}{M_1(t_1, \ldots, t_q)} dt_1 \ldots dt_q
$$

$$
< \left\| \phi_0 \right\| + \left( \int_{R_q} \phi_0 \right)^{1/p} \left( \sum_{q=1}^{\infty} \frac{\int_{R_q} M_1(t_1, \ldots, t_q) dt_1 \ldots dt_q}{M_1(t_1, \ldots, t_q)} \right)^{1/q} < \infty.
$$

THEOREM 3.4. The Fourier transform is a linear continuous transformation on $r_p^{SB}$ to $r_p^{SB}$.

PROOF. Since $r_p^{SB} = \bigcup_{s > 1} r_p^{s'B}$, we consider the Fourier transform on the space, $r_p^{s'B}$, to the space, $r_p^{s'B}$, where $s' > s$. We have for any norm $||| \cdot |||_{s'B_m}$, the following,

$$
||| \mathcal{F} \phi |||_{s'B_m} = \frac{\int_{R_q} \exp\left[-2\pi i (t_1 w_1 + \ldots + t_q w_q)\right] \phi(t_1, \ldots, t_q) dt_1 \ldots dt_q |||_{m!^{1/p}}}{(s'B_m)^q}
$$

\[ \begin{align*}
\frac{\sup_{q} \int_{\mathbb{R}^{q}} \exp(-2\pi i(t_{1}\omega_{1} + \cdots + t_{q}\omega_{q})) \phi(t_{1}, \ldots, t_{q}) dt_{1} \cdots dt_{q}}{q!^{1/p}}
&= \sup_{q} \sup_{m} M_{m}(\omega_{1}, \ldots, \omega_{q})
\quad \text{for } 0 < \alpha_{1} < m
\quad \text{and } 1 < i < q
\quad \text{where } (\omega_{1}, \ldots, \omega_{q}) \in \mathbb{R}^{q}
\end{align*} \]
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\[ \sup_{q} \frac{M_{2m+1}(t_{1}, \ldots, t_{q})}{(s^{B_{2m+1}})^{q}} \left( \frac{(s^{B_{2m+1}})^{q}}{(s^{B_{m}})^{q}} \right)^{1/p} \]

\[ 0 < a_{i} < q \]

\[ 1 < i < q \]

\[ (t_{1}, \ldots, t_{q}) \in R^{q} \]

\[ \left( \frac{\| \phi \|_{2m+1}}{(s^{B_{2m+1}})^{q}} \right)^{1/p} \frac{(s^{B_{2m+1}})^{q}}{(s^{B_{m}})^{q}}. \]  \hspace{1cm} (3.5)

Noting that \( B_{2m+1} < B_{m} \) and \( s' > s \) implies expression (3.5) is finite.

4. THE FOURIER TRANSFORM ON \((I_{-m}^{P,sB})\),

In a previous paper [9], it was shown that the dual of \((I_{-m}^{P,sB})\) denoted \((I_{-m}^{P,sB})'\) is the union of sets of the form,

\[ (I_{-m}^{P,sB}) = \{(F_{0}, F_{1}, \ldots, F_{q}) : F_{0} \in C \}. \] \hspace{1cm} (4.1)

\[ F_{q} \in S_{R} + (R^{q}), \quad \sum_{q=0}^{\infty} \left\| F_{q} \right\|_{-m}(s^{B_{m}})^{q} q^{-1/p} < \infty. \]

The generalized Fock dual functionals described in expression (4.1) can also be considered as sequences where the \( F_{q} \) are symmetric tempered distributions all having rank \( < m \). We also note if \( \phi \in I_{-m}^{P,sB} \) and \( F \in (I_{-m}^{P,sB})' \), then the evaluation of \( F \) at \( \phi \) is denoted as

\[ \langle \langle F, \phi \rangle \rangle = \sum_{q=0}^{\infty} \left\| F_{q} \right\|_{-m}(s^{B_{m}})^{q} q^{-1/p} < \infty. \] \hspace{1cm} (4.2)

EXAMPLE. 4.3 All the sets, \((I_{-m}^{P,sB})\), contain the generalized Fock Dirac functional,

\[ \delta \in I_{-m}^{P,sB} \]

\[ \left[ \begin{array}{c}
1 \\
\delta \\
\delta \delta \\
\vdots \\
\delta \delta \delta \ldots \delta
\end{array} \right] \] \hspace{1cm} (4.3)

where \( \delta \odot \delta \ldots \odot \delta \) is the tensor product of \( q \) copies of the Dirac delta functional [3]. We immediately verify that

\[ \left\| \delta \right\|_{-m}(s^{B_{m}})^{q} q^{-1/p} \]

\[ = \sum_{q=0}^{\infty} \left\| \delta \odot \ldots \odot \delta \right\|_{-m}(s^{B_{m}})^{q} q^{-1/p} \]

\[ < \sum_{q=0}^{\infty} 1 \cdot (s^{B_{m}})^{q} q^{-1/p} < \infty. \]
DEFINITION 4.4. The Fourier transform on the space $\mathcal{F}^B$, is defined as
\[ \langle \xi, \phi \rangle \triangleright \triangleleft \mathcal{F}^B \triangleright \triangleleft \langle \xi, \phi \rangle. \]

EXAMPLE 4.4. We compute the Fourier transform of
\[ \delta^k(t - \tau) \leftrightarrow \begin{bmatrix} 1 \\ \delta^k(t_1 - \tau_1) \\ \vdots \\ \delta^k(t_{q-1} - \tau_{q-1}) \end{bmatrix} \]
\[ \delta^k(t_q - \tau_q) \quad (4.4) \]

It suffices to consider the $q^{th}$ component,
\[ \langle \xi, \delta^k(t_1 - \tau_1) \otimes \cdots \otimes \delta^k(t_q - \tau_q) \rangle, \phi(w_1, \ldots, w_q) \]
\[ \langle \xi, \delta^k(t_1 - \tau_1) \otimes \cdots \otimes \delta^k(t_q - \tau_q) \rangle, \mathcal{F} \phi(w_1, \ldots, w_q) \]
\[ = \langle \delta^k(t_1 - \tau_1) \otimes \cdots \otimes \delta^k(t_q - \tau_q), \int_{R^q} \exp[-2\pi i (w_1 t_1 + \cdots + w_q t_q)] \phi(w_1, \ldots, w_q) \]
\[ \cdot dw_1 \cdots dw_q \rangle \]
\[ = \langle \delta^k(t_1 - \tau_1) \otimes \cdots \otimes \delta^k(t_q - \tau_q), \int_{R^q} \exp[-2\pi i (2\pi i w_1 t_1 + \cdots + 2\pi i w_q t_q)] \phi(w_1, \ldots, w_q) \]
\[ \cdot dw_1 \cdots dw_q \rangle \quad (4.5) \]

where $(2\pi i w)^k$ is being considered as a regular tempered distribution. In summary we have
\[ \langle \delta^k(t - \tau) \rangle \leftrightarrow \begin{bmatrix} 1 \\ (2\pi i w_1)^k e^{-2\pi i w_1 \tau_1} \\ \vdots \\ (2\pi i w_q)^k e^{-2\pi i w_q \tau_q} \end{bmatrix} \]
\[ (2\pi i w_1)^k (2\pi i w_2)^k \cdots (2\pi i w_q)^k e^{-2\pi i [w_1 \tau_1 + \cdots + w_q \tau_q]} \quad (4.6) \]
It is clear that any q'th entry in expression (4.6) does not belong to $L^2(\mathbb{R}^q)$ since clearly

$$\left| (2\pi i w_1)^k \ldots (2\pi i w_q)^k e^{-2\pi i [\sum_{1}^{q} w_i t_i + \ldots + q w_q t_q]} \right| =$$

and $$(2\pi i w_1)^k \ldots (2\pi i w_q)^k$$ is not integrable over $\mathbb{R}^q$.

However, the expression given in line (4.5) does belong to the $(\Gamma^p_{-m}, s^B_m)$ space since

$$\left| \sum_{q=0}^{\infty} \| F \|_{-m(s^B_m)^q} q!^{-1/p} \right|$$

$$= \sum_{q=0}^{\infty} \left| (2\pi i w_1)^k \ldots (2\pi i w_q)^k e^{-2\pi i [\sum_{1}^{q} w_i t_i + \ldots + q w_q t_q]} \right|_{-m(s^B_m)^q} q!^{-1/p}$$

$$< \sum_{q=0}^{\infty} 1(s^B_m)^q q!^{-1/p} < \infty.$$

EXAMPLE 4.7. In a similar computation it can be shown that

$${\mathcal F} : \begin{bmatrix} 1 \\ \delta \\ \vdots \\ \delta \circ \delta \circ \ldots \circ \delta \\ \vdots \\ \vdots \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \\ \vdots \end{bmatrix}$$

and again the Fourier transform is a member of every set, $(\Gamma^p_{-m}, s^B_m)$. It should be noted that other spaces such as distributions of exponential growth [3] offer some technical achievements that increase the space of Fourier transformable functions. However, we wanted to relate our results to our specialized scales of Fréchet spaces developed in Schmeelk [7-10] and Schwartz [15].

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This subject has been extensively studied in the past years for one-, two-, and three-dimensional space. Additionally, such dynamical systems can exhibit a very important and still unexplained phenomenon, called as the Fermi acceleration phenomenon. Basically, the phenomenon of Fermi acceleration (FA) is a process in which a classical particle can acquire unbounded energy from collisions with a heavy moving wall. This phenomenon was originally proposed by Enrico Fermi in 1949 as a possible explanation of the origin of the large energies of the cosmic particles. His original model was then modified and considered under different approaches and using many versions. Moreover, applications of FA have been of a large broad interest in many different fields of science including plasma physics, astrophysics, atomic physics, optics, and time-dependent billiard problems and they are useful for controlling chaos in Engineering and dynamical systems exhibiting chaos (both conservative and dissipative chaos).

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