EFFECT OF ROTATION AND RELAXATION TIMES ON PLANE WAVES IN GENERALIZED THERMO-VISCO-ELASTICITY

S. K. ROYCHOUDHURI and SANTWANA MUKHOPADHYAY

(Received 8 March 1996)

Abstract. The generalized dynamical theory of thermo-elasticity proposed by Green and Lindsay is applied to study the propagation of harmonically time-dependent thermo-visco-elastic plane waves of assigned frequency in an infinite visco-elastic solid of Kelvin-Voigt type, when the entire medium rotates with a uniform angular velocity. A more general dispersion equation is deduced to determine the effects of rotation, visco-elasticity, and relaxation time on the phase-velocity of the coupled waves. The solutions for the phase velocity and attenuation coefficient are obtained for small thermo-elastic couplings by the perturbation technique. Taking an appropriate material, the numerical values of the phase velocity of the waves are computed and the results are shown graphically to illustrate the problem.

Keywords and phrases. Plane waves, rotating visco-elastic medium, generalized thermo-elasticity.

2000 Mathematics Subject Classification. Primary 74Dxx.

1. Introduction. The classical theory of thermoelasticity is based on Fourier's law of heat conduction, which predicts an infinite speed of heat propagation. Many new theories have been proposed to eliminate this physical absurdity. Lord and Shulman [4] first modified Fourier's law by introducing into the field equations the term representing the thermal relaxation time. This modified theory is known as the generalized theory of thermoelasticity. Following Lord-Shulman's theory, several authors including Puri [7] and Nayfeh [6] studied the plane thermoelastic wave propagations. Later, Green and Lindsay [3] developed a more general theory of thermoelasticity, in which Fourier's law of heat conduction is unchanged, whereas the classical energy equation and the stress-strain temperature relations are modified by introducing two constitutive constants $\alpha$ and $\alpha^*$ having the dimensions of time. Using this theory, Agarwal [1, 2] considered, respectively, thermoelastic and magneto-thermoelastic plane wave propagation in an infinite elastic medium. Later, Mukhopadhyay and Bera [5] applied the generalized dynamical theory of thermoelasticity to determine the distributions of temperature, deformation, stress and strain in an infinite isotropic visco-elastic solid of Kelvin-Voigt type permeated by uniform magnetic field having distributed instantaneous and continuous sources.

Recently, attention has been given to the propagation of thermoelastic plane waves in a rotating medium. Following Lord-Shulman's theory, Puri [8], and Roychoudhuri and Debnath [10] studied plane wave propagation in infinite rotating elastic medium. Roychoudhuri [9] applied Green-Lindsay's theory to study the effect of rotation and

In the present paper, the linearized theory of Green and Lindsay having two relaxation times is applied to study the propagation of harmonically time-dependent thermo-visco-elastic plane waves of assigned frequency in an infinite rotating visco-elastic solid of Kelvin-Voigt type. Using the perturbation technique, a dispersion relation for small thermoelastic coupling is obtained to determine the effects of rotation and relaxation times on the phase velocity of the waves in a visco-elastic medium. Numerical values of the wave speeds at various frequencies are computed for an appropriate material and are presented graphically for the purpose of illustration.

2. Formulation of the problem. We consider an infinite isotropic homogeneous visco-elastic solid Kelvin-Voigt type which is rotating uniformly with an angular velocity \( \Omega \). The basic field equations in the temperature-rate dependent theory of Green and Lindsay follow (in usual notations).

(i) The stress equations of motion in a rotating medium in the absence of body forces are [12]

\[
\tau_{ij,j} = \rho \left\{ \ddot{u}_i + [\Omega \times (\Omega \times u)]_i + (2\Omega \times \dot{u})_i \right\}, \quad i, j = 1, 2, 3, \quad (2.1)
\]

where

\[
\tau_{ij} = \left( \lambda_e + \lambda_v \frac{\partial}{\partial t} \right) \Delta \delta_{ij} + 2 \left( \mu_e + \mu_v \frac{\partial}{\partial t} \right) e_{ij} - \gamma (\dot{\theta} + \alpha \dot{\theta}) \delta_{ij}. \quad (2.2)
\]

(ii) The heat conduction equation is

\[
K \theta_{ii,i} = \rho c_v \left( \dot{\theta} + \alpha^* \ddot{\theta} \right) + y \theta \dot{\Delta}, \quad i = 1, 2, 3. \quad (2.3)
\]

3. Plane wave solutions and dispersion relation. We consider the waves propagating in the \( x \)-direction and all the field variables are assumed to be functions of \( x \) and time \( t \) only. We assume that \( u = (u, v, w) \) and \( \Omega = (0, 0, \Omega) \), where \( \Omega \) is a constant. Equation (2.1) with equation (2.2) then reduces to

\[
\rho \left[ \ddot{u} - u \Omega^2 - 2 \dot{\nu} \Omega \right] = \left[ (\lambda_e + 2\mu_e) + (\lambda_v + 2\mu_v) \frac{\partial}{\partial t} \right] \frac{\partial^2 u}{\partial x^2} - \gamma \left( \frac{\partial \theta}{\partial x} + \alpha \frac{\partial^2 \theta}{\partial x \partial t} \right), \quad (3.1)
\]

\[
\rho \left[ \ddot{v} - \nu \Omega^2 + 2 \dot{u} \Omega \right] = \left( \mu_e + \mu_v \frac{\partial}{\partial t} \right) \frac{\partial^2 v}{\partial x^2}, \quad (3.2)
\]

\[
\rho \ddot{w} = \left( \mu_e + \mu_v \frac{\partial}{\partial t} \right) \frac{\partial^2 w}{\partial x^2}. \quad (3.3)
\]

The nondimensional forms of equations (3.1), (3.2), (3.3), and (2.3) are obtained as

\[
\dot{\theta} + \alpha^* \ddot{\theta} - \theta'' + \epsilon_u \dot{u}' = 0, \quad (3.4)
\]

\[
\left[ \ddot{u} - u \Omega^2 - 2 \dot{\nu} \Omega \right] = \left[ 1 + M \frac{\partial}{\partial t} \right] u'' - (\theta' + \alpha^* \dot{\theta}'), \quad (3.5)
\]

\[
\beta^2 \left[ \ddot{v} - \nu \Omega^2 + 2 \dot{u} \Omega \right] = \left( 1 + N \frac{\partial}{\partial t} \right) v'', \quad (3.6)
\]

\[
\beta^2 \ddot{w} = \left( 1 + N \frac{\partial}{\partial t} \right) w'', \quad (3.7)
\]
where we use the following notation and nondimensional variables:

\[
\begin{align*}
\c_1^2 &= \frac{\lambda_e + 2\mu_e}{\rho}, \quad \kappa = \frac{K}{\rho c_v}, \quad \epsilon_\theta = \frac{y^2 \rho^*}{\rho^2 c_v c_1^2}, \\
\beta^2 &= \frac{\lambda_e + 2\mu_e}{\mu_e}, \quad M = \frac{(\lambda_e + 2\mu_e)}{(\lambda_e + 2\mu_e)} \cdot \frac{c_1^2}{\kappa}, \\
N &= \frac{\mu_e}{\mu_e} \cdot \frac{c_1^2}{\kappa}, \quad \alpha^\prime = \frac{\alpha c_1^2}{\kappa}, \quad \alpha^{\prime \prime} = \frac{\alpha^* c_1^2}{\kappa}.
\end{align*}
\]

\(\kappa/c_1^2, \kappa/c_1, y\theta^* \kappa/c_1^3 \rho, \theta^*, c_1^2/\kappa\) are taken as the units of time, length, displacement, temperature, and rotation, respectively. Primes denote the differentiation with respect to \(x\) and dots denote time differentiation. Equations (3.4), (3.5), and (3.6) form a coupled system and represent coupled visco-thermal-dilatational and shear waves, while equation (3.7) un couples from the system. This coupling disappears when \(\Omega = 0\). Thus, the thermal field affects the dilatational and shear motion due to rotation.

4. Dispersion equation for the system. For harmonic solutions of the equations (3.4), (3.5), and (3.6), we choose

\[
(u, \nu, \theta) = (u_0, \nu_0, \theta_0) \cdot \exp \left\{ i(qx + \omega t) \right\},
\]

where \(u_0, \nu_0, \theta_0\) are amplitude constants, \(\omega\) is the prescribed frequency, \(q\) is the wave number, in general complex. The phase velocity \(c\) and the attenuation coefficients \(a\) are then given by

\[
c = \frac{\omega}{\Re(q)}, \quad a = -\Im(q).
\]

Substituting (4.1) into (3.4), (3.5), and (3.6), we obtain

\[
\begin{align*}
(A_1 q^2 - \omega^2 - \Omega^2) u_0 - 2i\omega \Omega \nu_0 - \alpha_1^* \omega q \theta_0 &= 0, \\
2i\omega \Omega \beta^2 u_0 + (A_2 q^2 - \omega^2 \beta^2 - \Omega^2 \beta^2) \nu_0 &= 0, \\
-\epsilon_\theta \omega q u_0 + (q^2 - \omega^2 \alpha_2^*) \theta_0 &= 0,
\end{align*}
\]

where

\[
\begin{align*}
\alpha_1^* &= \alpha' - \frac{i}{\omega}, \quad \alpha_2^* &= \alpha^{\prime \prime} - \frac{i}{\omega}, \\
A_1 &= 1 + i\omega M, \quad A_2 = 1 + i\omega N.
\end{align*}
\]

For the nontrivial solutions for \(u_0, \nu_0, \theta_0\), the dispersion equation of the coupled wave is obtained from (4.3) as

\[
(q^2 - \omega^2 \alpha_2^*) \left\{ (A_1 q^2 - \Omega_0^2) A_2 q^2 (A_2 q^2 - \Omega_0^2 \beta^2) - 4\Omega^2 \omega^2 \beta^2 \right\} - \epsilon_\theta \alpha_1^* \omega^2 q^2 (A_2 q^2 - \Omega_0^2 \beta^2) = 0,
\]

(4.5)

where \(\Omega_0^2 = \Omega^2 + \omega^2\). In case \(\Omega = 0\), the dispersion equation (4.5) reduces to

\[
(A_2 q^2 - \omega^2 \beta^2) \left\{ (q^2 - \omega^2 \alpha_2^*) (A_1 q^2 - \omega^2) - \epsilon_\theta \alpha_1^* \omega^2 q^2 \right\} = 0.
\]

(4.6)
On setting \( M = 0 = N \), equations (4.5) and (4.6) agree with [9, equations (3.7) and (3.8)], respectively. Equation (4.5) is therefore a more general dispersion equation in the sense that it incorporates the visco-elastic effect as well as the effects of rotation and relaxation parameters on the propagation of coupled waves. This wave may be called the quasi-visco-elastic-thermal-dilatational-shear wave.

5. Perturbation solution for small \( \epsilon_\theta \). To obtain the perturbation solution of the dispersion equation for small values of \( \epsilon_\theta \), we first put \( \epsilon_\theta = 0 \) in (4.5) to obtain the following solutions:

\[
q^2 = \omega^2 \alpha^*_2, \quad q^2_{\omega,1} = J^2_{\omega,1}, \tag{5.1}
\]

where

\[
J^2_{\omega,1} = \frac{(A_2 + A_1 \beta^2)\Omega^2_0 \pm \{(A_2 + A_1 \beta^2)^2\Omega^4_0 - 4A_1A_2\beta^2(\Omega^2 - \omega^2)^2\}^{1/2}}{2A_1A_2}. \tag{5.2}
\]

Next, let us write \( q^2 \) in the following forms:

\[
q^2_u = q^2_{\omega,1} + n_u \epsilon_\theta + 0(\epsilon^2_\theta), \quad q^2_\nu = q^2_{\omega,2} + n_\nu \epsilon_\theta + 0(\epsilon^2_\theta), \quad q^2_\theta = \omega^2 \alpha^*_2 + n_\theta \epsilon_\theta + 0(\epsilon^2_\theta). \tag{5.3}
\]

Substituting into equation (4.5), comparing the lowest power of \( \epsilon_\theta \), and neglecting the terms of \( 0(\epsilon^2_\theta) \), we obtain

\[
q^2_u = J^2_u \left[ 1 + \frac{\omega^2 \alpha^*_1 \epsilon_\theta(A_2J^2_{\omega,1} - \beta^2\Omega^2_0)}{D_1} \right], \tag{5.4}
\]

\[
q^2_\nu = J^2_\nu \left[ 1 + \frac{\omega^2 \alpha^*_1 \epsilon_\theta(A_2J^2_{\omega,2} - \beta^2\Omega^2_0)}{D_2} \right], \tag{5.5}
\]

\[
q^2_\theta = \omega^2 \alpha^*_2 \left[ 1 + \frac{\alpha^*_\theta \omega^2 \epsilon_\theta(A_2\alpha^*_2 \omega^2 - \beta^2\Omega^2_0)}{(A_1 \alpha^*_2 \omega^2 - \Omega^2_0)(A_2 \alpha^*_2 \omega^2 - \beta^2\Omega^2_0) - 4\Omega^2 \beta^2 \omega^2} \right], \tag{5.6}
\]

where

\[
D_{1,2} = (J^2_{\omega,1} - \omega^2 \alpha^*_2) \left[ 2A_1A_2J^2_{\omega,2} - A_1 \beta^2 \Omega^2_0 - A_2 \Omega^2_0 \right] + (A_1J^2_{\omega,1} - \Omega^2_0)(A_2J^2_{\omega,2} - \beta^2\Omega^2_0) - 4\Omega^2 \beta^2 \omega^2. \tag{5.7}
\]

On putting \( M = 0 = N \), the results (5.2), (5.4), and (5.5) are in agreement with the corresponding results of [9].

6. Determination of wave speeds and attenuation coefficients. From the above solutions, we can observe that the dilatational, shear, and thermal waves propagate in the visco-elastic medium, and these waves are affected by the thermo-visco-elastic coupling coefficient \( \epsilon_\theta \). Now we find out the wave speed and the attenuation coefficients of the waves for small \( \epsilon_\theta \).
I. Quasi-visco-thermal wave. Separating real and imaginary parts, we get from (5.6), for small $\epsilon_\theta$,

\[
\text{Re}(q_\theta) = \omega \sqrt{\frac{L + \alpha'''}{2} + \frac{1}{2} \epsilon_\theta \cdot \frac{\omega^3}{\sqrt{2}} \cdot \frac{1}{(D_2^2 + D_m^2)}} \times \left[ \sqrt{L + \alpha'''}(N_y D_y + N_m D_m) - \sqrt{L - \alpha'''}(N_m D_y - N_y D_m) \right], \\
\text{Im}(q_\theta) = \omega \sqrt{\frac{L - \alpha'''}{2} + \frac{1}{2} \epsilon_\theta \cdot \frac{\omega^3}{\sqrt{2}} \cdot \frac{1}{(D_2^2 + D_m^2)}} \times \left[ \sqrt{L + \alpha'''}(N_m D_y - N_y D_m) - \sqrt{L - \alpha'''}(N_y D_y + N_m D_m) \right],
\]

where

\[
N_y = \alpha'(\alpha''' \omega^2 + N \omega^2 - \Omega_0^2 \beta^2) - (1 - \alpha''' N \omega^2), \\
N_m = -\left( \alpha''' \omega + N \omega - \frac{\Omega_0^2 \beta^2}{\omega} \right) - \alpha' \omega (1 - \alpha''' N \omega^2), \\
D_y = (\alpha''' \omega^2 + N \omega^2 - \Omega_0^2) (\alpha''' \omega^2 + N \omega^2 - \Omega_0^2 \beta^2) \\
- \omega^2 (1 - \alpha''' M \omega^2) (1 - \alpha''' N \omega^2) - 4 \omega^2 \Omega_0^2 \beta^2, \\
D_m = -\omega \left[ (1 - \alpha''' M \omega^2) (\alpha''' \omega^2 + N \omega^2 - \Omega_0^2 \beta^2) \\
+ (1 - \alpha''' N \omega^2) (\alpha''' \omega^2 + M \omega^2 - \Omega_0^2) \right],
\]

\[
L = \sqrt{\left[ (\alpha''' \omega)^2 + 1 \right]}, \\
\sqrt{\alpha''^2} = \frac{1}{\sqrt{2}} \left[ \sqrt{L + \alpha''} + i \sqrt{L - \alpha''} \right].
\]

Therefore, the thermal wave speed $c_\theta = \omega \text{Re}(q_\theta)$ and the attenuation coefficient $a_\theta = -\text{Im}(q_\theta)$, where $\text{Re}(q_\theta)$ and $\text{Im}(q_\theta)$ are obtained above.

II. Quasi-visco-dilatational wave. Using (5.2) from (5.4) for small $\epsilon_\theta$, the quasi-visco-elastic dilatational wave speed $c_e = \omega \text{Re}(q_u)$ and the attenuation coefficient $a_e = -\text{Im}(q_u)$, where

\[
\text{Re}(q_u) = A_3 + \frac{1}{2} \epsilon_\theta \omega \cdot \frac{1}{(P^2 + Q^2)} \left[ A_3 \left[ P(\alpha' \omega K_3 + K_4) + Q(\alpha' \omega K_4 - K_3) \right] \\
- B_3 \left[ P(\alpha' \omega K_4 - K_3) - Q(\alpha' \omega K_3 + K_4) \right] \right], \\
\text{Im}(q_u) = B_3 + \frac{1}{2} \epsilon_\theta \omega \cdot \frac{1}{(P^2 + Q^2)} \left[ B_3 \left[ P(\alpha' \omega K_3 + K_4) + Q(\alpha' \omega K_4 - K_3) \right] \\
+ A_3 \left[ P(\alpha' \omega K_4 - K_3) - Q(\alpha' \omega K_3 + K_4) \right] \right],
\]

(6.5)
where

\[
A_3 = \frac{1}{2} \left[ R_3 + \sqrt{R_3^2 + R_4^2} \right]^{1/2}, \quad B_3 = \frac{1}{2} \left( \frac{R_4}{|R_4|} \right) \left[-R_3 + \sqrt{R_3^2 + R_4^2} \right]^{1/2},
\]

\[
R_3 = \frac{\left[ (1 - \omega^2 MN) \left( (\beta^2 + 1) \Omega_0^2 + A \right) + (M + N) \omega \times \left\{ \Omega_0^2 \omega (N + M \beta^2) - B \right\} \right]}{\left[ (1 - \omega^2 MN)^2 + \omega^2 (M + N)^2 \right]},
\]

\[
R_4 = \frac{\left[ (1 - \omega^2 MN) \left( (\beta^2 + 1) \Omega_0^2 - A \right) - (M + N) \omega \times \left\{ (\beta^2 + 1) \Omega_0^2 - B \right\} \right]}{\left[ (1 - \omega^2 MN)^2 + \omega^2 (M + N)^2 \right]},
\]

\[
A = \frac{1}{\sqrt{2}} \left[ P_1 + \sqrt{P_1^2 + P_2^2} \right]^{1/2}, \quad B = \frac{P_2}{|P_2|} \cdot \frac{1}{\sqrt{2}} \left[ -P_1 + \sqrt{P_1^2 + P_2^2} \right]^{1/2},
\]

\[
P_1 = \left( (\beta^2 + 1)^2 - \omega^2 (N + M \beta^2) \right) \Omega_0^2 - 4 \beta^2 (1 - \omega^2 MN) (\Omega^2 - \omega^2),
\]

\[
P_2 = \omega \left[ 2 (\beta^2 + 1) (N + M \beta^2) \Omega_0^2 - 4 \beta^2 (M + N) (\Omega^2 - \omega^2) \right],
\]

\[
K_3 = R_3 - \beta^2 \Omega_0^2 - R_4 \omega M, \quad K_4 = R_4 + R_3 \omega M,
\]

\[
E_3 = 2 R_3 (1 - \omega^2 MN) - 2 R_4 \omega (M + N) - (\beta^2 + 1) \Omega_0^2,
\]

\[
E_4 = 2 R_4 (1 - \omega^2 MN) + 2 R_3 \omega (M + N) - \omega (N + M \beta^2) \Omega_0^2,
\]

\[
F_3 = R_3 - R_4 \omega N - 2 \Omega_0^2, \quad F_4 = R_4 + R_3 \omega N,
\]

\[
P = E_3 (R_3 - \alpha^{*} \omega^2) - E_4 (R_4 + \omega) + F_3 K_3 - F_4 K_4 - 4 \Omega^2 \omega^2 \beta^2,
\]

\[
Q = E_3 (R_3 + \omega) + E_4 (R_3 - \alpha^{*} \omega^2) + F_4 K_3 + F_3 K_4.
\]

III. QUASI-VISCO-SHEAR WAVE. Using (5.2), we get from (5.5) for small \(\epsilon_0\), the quasi-visco-shear wave speed \(c_5 = \omega / \text{Re}(q_\nu)\) and the attenuation coefficient \(a_5 = -\text{Im}(q_\nu)\), where

\[
\text{Re}(q_\nu) = A_1 + \frac{1}{2} \epsilon_0 \omega \cdot \frac{1}{(P^{*2} + Q^{*2})} \left[ A_1 \left\{ P' (\alpha' \omega K_1 + K_2) \right\} \right. \\
\left. + Q' (\alpha' \omega K_2 - K_1) \right] - B_1 \left[ P' (\alpha' \omega K_2 - K_1) - Q' (\alpha' \omega K_1 + K_2) \right],
\]

\[
\text{Im}(q_\nu) = B_1 + \frac{1}{2} \epsilon_0 \omega \cdot \frac{1}{(P^{*2} + Q^{*2})} \left[ B_1 \left\{ P' (\alpha' \omega K_1 + K_2) \right\} + Q' (\alpha' \omega K_2 - K_1) \right] + A_1 \left[ P' (\alpha' \omega K_2 - K_1) - Q' (\alpha' \omega K_1 + K_2) \right],
\]

\[
A_1 = \frac{1}{2} \left[ R_1 + \sqrt{R_1^2 + R_2^2} \right]^{1/2}, \quad B_1 = \frac{R_2}{|R_2|} \cdot \frac{1}{2} \left[-R_1 + \sqrt{R_1^2 + R_2^2} \right]^{1/2},
\]

\[
P' = E_1 (R_1 - \alpha^{*} \omega^2) - E_2 (R_2 + \omega) + F_1 K_1 - F_2 K_2 - 4 \Omega^2 \omega^2 \beta^2,
\]

\[
Q' = E_1 (R_2 + \omega) + E_2 (R_1 - \alpha^{*} \omega^2) + F_2 K_1 + F_1 K_2,
\]

where \(R_1\) and \(R_2\) have the same expressions as \(R_3\) and \(R_4\), respectively, with \(-A, -B\) in place of \(A, B\), respectively, and \(E_1, E_2, K_1, K_2, F_1, F_2\) have the same expressions as \(E_3, E_4, K_3, K_4, F_3, F_4\), respectively, with \(R_1\) and \(R_2\) in place of \(R_3\) and \(R_4\), respectively.
7. Numerical results and discussions. For numerical work, we take copper as the working substance for which

\[
\begin{align*}
\lambda_e &= 1.387 \times 10^{12} \text{ dyne/cm}^2, \\
\mu_e &= 0.448 \times 10^{12} \text{ dyne/cm}^2, \\
\rho &= 8.93 \text{ g/cm}^3, \\
\kappa &= 1.14 \text{ cm}^2/\text{s}, \\
\alpha &= 1.67 \times 10^{-8}/\circ \text{c}, \\
\sigma &= 10^{-11} \text{ s}, \\
\sigma^* &= 2 \times 10^{-11} \text{ s}.
\end{align*}
\] (7.1)

We take \( \Omega = 0.1 \).

The numerical computations of the quasi-visco-dilatational wave speed, the quasi-visco-shear wave speed, and the quasi-visco-thermal wave speed for small values of \( \epsilon_\theta \) are done with the help of a PC and the corresponding graphs are plotted.

The quasi-visco-dilatational wave speed is drawn against the real frequency \( \omega \) for three different values of the visco-elastic parameter \( M \) in Figure 7.1. The quasi-visco-shear wave speed is plotted against \( \omega \) for three different values of \( M \) in Figure 7.2 and the quasi-visco-thermal wave speed is drawn against \( \omega \) for the same values of \( M \) in Figure 7.3.

The variation of different wave speeds with the visco-elastic parameter \( M \) can be seen from the graphs. It is observed that although in case of quasi-visco-thermal wave speed (Figure 7.3) the variation is not pronounced appreciably, in cases of the quasi-visco-dilatational wave speed (Figure 7.1) and the quasi-visco-thermal-shear wave speed (Figure 7.2) the variations are more pronounced for higher values of \( \omega \).
Figure 7.2. Plot of quasi-visco-shear wave speed against real frequency $\omega$.

Figure 7.3. Plot of quasi-visco-thermal wave speed against real frequency $\omega$.

Acknowledgement. The authors would like to offer grateful thanks to the Council of Scientific and Industrial Research (CSIR, India) for full financial support.
EFFECT OF ROTATION AND RELAXATION TIMES ON PLANE ... 505

REFERENCES


ROYCHOU DHURI: DEPARTMENT OF MATHEMATICS, UNIVERSITY OF BURDWAN, BURDWAN 713104, INDIA

MUKHOPADHYAY: DEPARTMENT OF APPLIED MATHEMATICS, INSTITUTE OF TECHNOLOGY, BANARAS HINDU UNIVERSITY, VARANASI-221 005, INDIA
Thinking about nonlinearity in engineering areas, up to the 70s, was focused on intentionally built nonlinear parts in order to improve the operational characteristics of a device or system. Keying, saturation, hysteretic phenomena, and dead zones were added to existing devices increasing their behavior diversity and precision. In this context, an intrinsic nonlinearity was treated just as a linear approximation, around equilibrium points.

Inspired on the rediscovering of the richness of nonlinear and chaotic phenomena, engineers started using analytical tools from “Qualitative Theory of Differential Equations,” allowing more precise analysis and synthesis, in order to produce new vital products and services. Bifurcation theory, dynamical systems and chaos started to be part of the mandatory set of tools for design engineers.

This proposed special edition of the Mathematical Problems in Engineering aims to provide a picture of the importance of the bifurcation theory, relating it with nonlinear and chaotic dynamics for natural and engineered systems. Ideas of how this dynamics can be captured through precisely tailored real and numerical experiments and understanding by the combination of specific tools that associate dynamical system theory and geometric tools in a very clever, sophisticated, and at the same time simple and unique analytical environment are the subject of this issue, allowing new methods to design high-precision devices and equipment.

Authors should follow the Mathematical Problems in Engineering manuscript format described at http://www.hindawi.com/journals/mpe/. Prospective authors should submit an electronic copy of their complete manuscript through the journal Manuscript Tracking System at http://mts.hindawi.com/ according to the following timetable:

<table>
<thead>
<tr>
<th>Event</th>
<th>Date</th>
</tr>
</thead>
<tbody>
<tr>
<td>Manuscript Due</td>
<td>December 1, 2008</td>
</tr>
<tr>
<td>First Round of Reviews</td>
<td>March 1, 2009</td>
</tr>
<tr>
<td>Publication Date</td>
<td>June 1, 2009</td>
</tr>
</tbody>
</table>

**Guest Editors**

José Roberto Castilho Piqueira, Telecommunication and Control Engineering Department, Polytechnic School, The University of São Paulo, 05508-970 São Paulo, Brazil; piqueira@lac.usp.br

Elbert E. Neher Macau, Laboratório Associado de Matemática Aplicada e Computação (LAC), Instituto Nacional de Pesquisas Espaciais (INPE), São José dos Campos, 12227-010 São Paulo, Brazil; elbert@lac.inpe.br

Celso Grebogi, Center for Applied Dynamics Research, King’s College, University of Aberdeen, Aberdeen AB24 3UE, UK; grebogi@abdn.ac.uk