LAMB'S PLANE PROBLEM IN A THERMO-ELASTIC MICROPOLAR MEDIUM WITH STRETCH

T.K. CHADHA and RAJNEESH KUMAR
Department of Mathematics
Guru Nanak Dev University
Amritsar, India

and

LOKENATH DEBNATH
Department of Mathematics
University of Central Florida
Orlando, Florida 32816, USA

(Received May 23, 1984 and in revised form January 8, 1985)

ABSTRACT. A study is made of the Lamb plane problem in a thermo-elastic micropolar medium with the effect of stretch. The problem is solved for an arbitrary, normal load distribution by using the double Fourier transform. The displacement components, force stress, couple stress, vector first moment and the temperature field are determined for a half space subjected to an arbitrary normal load. Two special cases of a horizontal force and a torque which are oscillating with a frequency $\omega$ have been investigated. It is shown that results of this analysis reduce to those without stretch.

KEY WORDS AND PHRASES. Lamb's problem, Fourier transform, and micropolar medium.

1980 MATHEMATICS SUBJECT CLASSIFICATION CODE. 78D30

1. INTRODUCTION.

Eringen and Suhubi [1] have developed a general theory of linear and nonlinear micro-elastic continua. This theory contains the Cosserat continuum and the intermediate couple stress theories as special cases. In a subsequent paper [2], Eringen recapitulated his work and renamed his theory as micropolar elasticity. The micropolar theory essentially deals with such materials whose constituents are dumbbell type molecules and are allowed to rotate independently without stretch. Later on, Eringen [3-4] extended his work to include the effect of axial stretch during the rotation of molecules and developed theories for both micropolar elastic solids with stretch and micropolar fluids with stretch. The mechanical model underlying the theory of micropolar elastic solids with stretch can be envisioned as an elastic medium composed of a large number of short springs. These springs possess average inertia and can deform in axial directions.
Lamb's problem [5] has been investigated extensively by several researchers in different elastic media with various kinds of loading. In particular, Nowacki and Nowacki [6] have studied the Lamb problem in micropolar elastic media. Recently, Chadha [7] has investigated the same problem in micropolar elastic media, and discussed wave propagation in a semi-infinite micropolar elastic solid due to loading at the plane boundary of semi-half space. Acharya and Sengupta [8] have recently studied Lamb's problem in a thermo-elastic medium under the influence of temperature. They have examined the longitudinal and transverse thermo-elastic wave propagation in a micropolar semi-infinite space bounded by a plane in which a normal loading is applied.

In spite of these studies, no attention is given to Lamb's problem in thermo-micropolar elastic half-space with stretch. The main purpose of this paper is to investigate the problem with the assumption that the heat is radiated from the free plane boundary surface of the semi-infinite space and the maximum temperature difference across the surface is always small. The displacement components, force stress, couple stress, vector first moment and the temperature field are determined for the half-space subjected to an arbitrary normal load. Two special cases of a horizontal force and a torque which are harmonic in time have been discussed. The problem is solved by the double Fourier transform method.

2. THE FORMULATION OF THE PROBLEM AND THE BOUNDARY CONDITIONS

We consider a homogeneous micropolar elastic semi-infinite space with stretch under the influence of temperature. We assume that there is a uniform stretch in the x-direction only and a loading \( g(x,t) \) normal to the free boundary surface \( z = 0 \). Further, we assume that the micropolar semi-space is free to exchange heat within the region \( z > 0 \); and prior to the appearance of any disturbance, both media are everywhere at the constant absolute temperature \( T_0 \).

We consider the two-dimensional problem so that the displacement and rotation are independent of the y coordinate. Thus we may write \( \mathbf{u} = (u_1, 0, u_3) \) and \( \mathbf{\omega} = (0, \omega_2, 0) \). The displacements are related to the displacement potentials \( \psi(x, z, t) \) and \( \varphi(x, z, t) \) as follows:

\[
\begin{align*}
    u_1 &= \frac{\partial \varphi}{\partial x} + \frac{\partial \psi}{\partial z}, &
    u_3 &= \frac{\partial \varphi}{\partial z} - \frac{\partial \psi}{\partial x} \\
    \end{align*}
\]

so that

\[
\begin{align*}
    e &= \nabla^2 \varphi, &
    \psi &= \frac{\partial u_1}{\partial z} - \frac{\partial u_3}{\partial x} \\
\end{align*}
\]

where

\[
\begin{align*}
    \psi^2 &= \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} &
    e &= \frac{\partial^2 u_1}{\partial x^2} + \frac{\partial^2 u_3}{\partial z^2}.
\end{align*}
\]

We follow Eringen [4] and Nowacki [9] to write down the basic field equations in a thermo-micropolar elastic solid medium with stretch and without body forces and body moments. These field equations are

\[
\begin{align*}
(u+\alpha)v^2 \mathbf{u} + (\lambda+\mu-\alpha) \text{grad div } \mathbf{u} + 2 \alpha \text{ rot } \mathbf{\omega} - \nu \text{ grad } \mathbf{e} &= \rho \frac{\partial^2 \mathbf{u}}{\partial t^2}, \\
(\gamma+\epsilon)v^2 \mathbf{\omega} + (\gamma+\beta-\epsilon) \text{grad div } \mathbf{\omega} - 4 \alpha \mathbf{\omega} + 2 \alpha \text{ rot } \mathbf{u} &= \mu \frac{\partial^2 \mathbf{\omega}}{\partial t^2},
\end{align*}
\]
where \( \lambda, \mu, \alpha, \beta, \gamma, \epsilon, \alpha_0, \eta_0 \) are material constants, \( \rho \) is the density of the material, \( J \) is the rotational inertia, \( \nu = (3\lambda + 2\mu) \alpha_t \), \( \alpha_t \) is the coefficient of linear expansion of the solid, \( \Theta = T - T_0 \) = absolute temperature minus the initial absolute temperature \( T_0 \).

Using the values of \( \nu \) and \( \omega \) in equations (2.4)-(2.6) we get

\[
(\mu + \alpha) \nu^2 u_1 + (\lambda + \mu - \alpha) \frac{\partial \nu}{\partial x} - 2\alpha \frac{\partial \omega}{\partial z} - \nu \frac{\partial \Theta}{\partial x} = \rho \frac{\partial^2 u_1}{\partial t^2},
\]

(2.7)

\[
(\mu + \alpha) \nu^2 u_3 + (\lambda + \mu - \alpha) \frac{\partial \nu}{\partial z} + 2\alpha \frac{\partial \omega}{\partial z} - \nu \frac{\partial \Theta}{\partial z} = \rho \frac{\partial^2 u_3}{\partial t^2},
\]

(2.8)

\[
(\gamma + \epsilon) \nu^2 \omega_2 - 4\alpha \omega_2 + 2\alpha \left( \frac{\partial u_1}{\partial z} - \frac{\partial u_3}{\partial x} \right) = J \frac{\partial^2 \omega_2}{\partial t^2},
\]

(2.9)

\[
\alpha_0 \nu^2 \phi - \eta_0 \phi = \frac{\partial^2 \phi}{\partial t^2}.
\]

(2.10)

The temperature field \( \theta(x,z,t) \) satisfies Fourier's Law of heat conduction, which in the present case can be written as

\[
\kappa \nabla^2 \theta = \rho C \frac{\partial^2 \theta}{\partial t^2} + T_0 \nu \frac{\partial}{\partial t} (\nabla^2 \phi),
\]

(2.11)

where \( \kappa \) is thermal conductivity and \( C \) is the specific heat at constant strain.

Using (2.1ab)-(2.3ab) and (2.11) in equations (2.7)-(2.10), we obtain

\[
(\nu^2 - \frac{1}{c_1^2} \frac{\partial^2}{\partial t^2}) \phi - \rho \theta = 0,
\]

(2.12)

\[
(\nu^2 - \frac{1}{c_2^2} \frac{\partial^2}{\partial t^2}) \theta - \rho \frac{\partial}{\partial t} (\nabla^2 \phi) = 0,
\]

(2.13)

\[
(\nu^2 - \frac{1}{c_2^2} \frac{\partial^2}{\partial t^2}) \psi - \rho \omega_2 = 0,
\]

(2.14)

\[
(\nu^2 - \gamma_1 \frac{1}{c_4^2} \frac{\partial^2}{\partial t^2}) \omega_2 + s \nu^2 \psi = 0,
\]

(2.15)

\[
(\nu^2 - \gamma_2 \frac{1}{c_5^2} \frac{\partial^2}{\partial t^2}) \phi = 0,
\]

(2.16)

where

\[
c_1^2 = \frac{\lambda + 2\mu}{\rho}, \quad c_2^2 = \frac{\mu + \alpha}{\rho}, \quad c_3^2 = \frac{K}{\rho C}, \quad c_4^2 = \frac{\gamma + \epsilon}{J}, \quad c_5^2 = \frac{2\alpha_0}{J} \quad (2.17abcde)\]
\[ \gamma_1^2 = \frac{4\alpha}{\gamma + \epsilon}, \gamma_2^2 = \frac{n_0}{\alpha_0}, p = \frac{2\alpha}{\mu + \alpha}, q = \frac{\nu}{\lambda + 2\mu}, \] (2.18abcd)

\[ r = \frac{\nu}{\kappa}, s = \frac{2\alpha}{\gamma + \epsilon}. \] (2.19ab)

We next eliminate \( \phi \) or \( \theta \) from equations (2.12)-(2.13), and \( \psi \) or \( \omega_2 \) from (2.14)-(2.15) to obtain the following partial differential equations:

\[ \left[ \left( \nu^2 - \frac{1}{c_1^2} \frac{\partial^2}{\partial t^2} \right) \left( \nu^2 - \frac{1}{c_3^2} \frac{\partial^2}{\partial t^2} \right) - \tau^2 \frac{\partial}{\partial t} \nu^2 \right] \left( \phi, \theta \right) = 0, \] (2.20)

\[ \left[ \left( \nu^2 - \frac{1}{c_2^2} \frac{\partial^2}{\partial t^2} \right) \left( \nu^2 - \frac{1}{c_4^2} \frac{\partial^2}{\partial t^2} \right) + \zeta^2 \nu^2 \right] \left( \psi, \omega_2 \right) = 0, \] (2.21)

where \( \tau^2 = q\epsilon, \zeta^2 = ps. \)

Following Eringen [4] and Nowacki [9] the stress tensor \( \sigma_{ij} \) and the couple stress tensor \( \mu_{ij} \) are given by

\[ \sigma_{ij} = (\lambda u_{ik} k - \nu \delta_{ij}) \delta_{ij} + (\mu - \alpha)(u_{ij} + u_{ji}) + 2\alpha(u_{ij} - \epsilon_{kji} u_{ik}), \] (2.22)

\[ \mu_{ij} = \beta_0 \epsilon_{kji} k + \beta w_{ik} k \delta_{ij} + (\gamma - \epsilon) w_{ij} + (\gamma + \epsilon) w_{ij}, \] (2.23)

\[ \beta_j = \alpha_0 \phi_{j} + \frac{1}{3} \beta_0 \epsilon_{kji} u_{ik}, \] (2.24)

where \( \epsilon_{kji} \) is unit antisymmetric tensor, \( \beta_j \) is the vector first moment and \( i, j, k = 1, 2, 3. \) These expressions in the present case reduce to the form

\[ \sigma_{33} = 2\nu \left( \frac{\partial^2 \phi}{\partial z^2} - \frac{\partial^2 \psi}{\partial x^2} \right) + \lambda \psi^2 \phi - \nu \theta, \] (2.25)

\[ \sigma_{31} = \nu \left[ \frac{\partial^2 \phi}{\partial z^2} + \frac{\partial^2 \psi}{\partial x^2} - \frac{\partial^2 \psi}{\partial x^2} \right] + \alpha(\psi^2 - 2\omega_2), \] (2.26)

\[ \mu_{32} = (\gamma + \epsilon) \frac{\partial \omega_2}{\partial z} - \beta_0 \frac{\partial \phi}{\partial x}, \] (2.27)

\[ \beta_3 = \alpha_0 \frac{\partial \phi}{\partial z} + \frac{1}{3} \beta_0 \frac{\partial \omega_2}{\partial x}. \] (2.28)

3. BOUNDARY CONDITIONS.

In view of the normal loading of magnitude \( g(x,t) \) applied on \( z = 0, \) the boundary conditions are given by

\[ \sigma_{33} = - g(x,t), \sigma_{31} = 0, \mu_{32} = 0, \beta_3 = 0, \text{ at } z = 0. \] (3.1abcd)

In view of the assumption that the temperature difference across the free surface is always small, the linearized form of the radiation condition is valid on the boundary \( z = 0 \) so that

\[ \frac{\partial \theta}{\partial z} + h \theta = 0 \quad \text{on } z = 0. \] (3.2)

Further, if we assume that the loading function \( g(x,t) \) is bounded and finite on \( z = 0, \) then \( \phi, \psi, \theta, \omega_2 \) and \( \phi \) vanish at infinity.
4. SOLUTION OF THE PROBLEM.

We solve the above equations (2.16), (2.20) and (2.21) by using the double Fourier transform defined as follows:

$$\tilde{f}(k,z,n) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,z,t) e^{i(kx+nt)} \, dx \, dt,$$

where the inverse transform is given by

$$f(x,z,t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \tilde{f}(k,z,n) e^{-i(kx+nt)} \, dk \, dn.$$

Thus the equations reduce to the form

$$\left(\frac{d^2}{dz^2} - \lambda_1^2\right)\left(\frac{d^2}{dz^2} - \lambda_2^2\right)(\tilde{\delta}, \tilde{\epsilon}) = 0,$$

$$\left(\frac{d^2}{dz^2} - \lambda_3^2\right)\left(\frac{d^2}{dz^2} - \lambda_4^2\right)(\tilde{\nu}, \tilde{\omega}_2) = 0,$$

$$\left(\frac{d^2}{dz^2} - \lambda_5^2\right)\phi = 0,$$

where

$$\lambda_1^2 + \lambda_2^2 = (2k^2 - \frac{n^2}{c_1} - \frac{n^2}{c_3} - \omega^2),$$

$$\lambda_3^2 + \lambda_4^2 = (2k^2 - \frac{n^2}{c_2} - \frac{n^2}{c_4} - \gamma^2),$$

$$\lambda_3^2 + \lambda_4^2 = (\frac{n^2}{c_2} - k^2)(\frac{n^2}{c_4} - k^2 - \gamma^2),$$

$$\lambda_5^2 = k^2 + \frac{n^2}{c_5}.$$

In view of the boundary conditions at infinity, the bounded solutions of (4.3)-(4.5) assume the form

$$\tilde{\delta} = A_1 e^{-\lambda_1 z} + B_1 e^{-\lambda_2 z},$$

$$\tilde{\epsilon} = A_2 e^{-\lambda_3 z} + B_2 e^{-\lambda_4 z},$$

$$\tilde{\nu} = C_1 e^{-\lambda_3 z} + D_1 e^{-\lambda_4 z},$$

$$\tilde{\omega}_2 = C_2 e^{-\lambda_3 z} + D_2 e^{-\lambda_4 z},$$

$$\phi = E_1 e^{-\lambda_5 z},$$

where

$$A_1 = a_1 A, B_1 = a_2 B, C_1 = a_3 C, D_1 = a_4 D,$$
and

\[
\alpha_j = \begin{cases} 
\frac{1}{\mu} \left( \lambda_j^2 + \frac{n^2}{c_1^2} - k^2 \right), & \text{for } j=1,2, \\
\frac{1}{\rho} \left( \lambda_j^2 + \frac{n^2}{c_2^2} - k^2 \right), & \text{for } j=3,4.
\end{cases}
\]  

(4.17ab)

It is assumed that \( \Re(\lambda_j) \geq 0, j=1,2,3,4,5. \)

Applying the Fourier transform (4.1) to (3.1abcd)-(3.2) and using (2.25)-(2.28), it turns out that

\[
2\mu \frac{d^2\bar{\psi}}{dz^2} + i k \bar{\bar{\psi}} \right) + \lambda \left( \frac{d^2}{dz^2} - k^2 \right) \bar{\psi} - \nu \bar{\theta} = - \bar{g}(k,n),
\]

(4.18)

\[
u(k + \frac{d^2}{dz^2} - 2ik \frac{d\bar{\psi}}{dz} + \alpha \left( \frac{d^2\bar{\psi}}{dz^2} - k^2 \bar{\psi} - 2\bar{\omega}_2 \right) = 0,
\]

(4.19)

\[(\gamma + \epsilon) \frac{d\bar{\omega}_2}{dz} + i \beta_0 \bar{\bar{\theta}} = 0,
\]

(4.20)

\[a_0 \frac{d\bar{\psi}}{dz} - \frac{1}{3} i \beta_0 \bar{\bar{\omega}}_2 = 0,
\]

(4.21)

\[\frac{d\bar{\vartheta}}{dz} + h \bar{\theta} = 0,
\]

(4.22)

where \( \bar{g}(k,n) \) is the double Fourier transform of \( g(x,t) \).

Substitution of (4.11) - (4.15) into (4.18) - (4.22) yields

\[q_1A + q_2B + q_3C + q_4D = - \bar{g}(k,n),
\]

(4.23)

\[p_1A + p_2B + p_3C + p_4D = 0,
\]

(4.24)

\[r_3C + r_4D + r_5E = 0,
\]

(4.25)

\[t_3C + t_4D + t_5E = 0,
\]

(4.26)

\[s_1A + s_2B = 0,
\]

(4.27)

where

\[q_j = \begin{cases} 
\lambda_j^2(\lambda + 2\nu) - k^2\lambda - \alpha_j\nu, & j=1,2, \\
-2i \mu k \lambda_j, & j=3,4.
\end{cases}
\]  

(4.28ab)

\[p_j = \begin{cases} 
-2i \mu k \lambda_j, & j=1,2, \\
\nu(k^2 + \lambda_j^2) + \alpha(\lambda_j^2 - k^2 - 2\lambda_j), & j=3,4.
\end{cases}
\]  

(4.29ab)
LAMB'S PLANE PROBLEM IN A THERMO-ELASTIC MICROPOLAR MEDIUM

\[ r_j = \begin{bmatrix} (r+\epsilon) a_j \lambda_j, \\ -i\delta_k, \end{bmatrix} \quad j = 3,4, \]  
\[ s_j = (\lambda_j - \eta) a_j, \quad j = 1,2, \]  
\[ t_j = \begin{bmatrix} i\delta_k a_j, \\ 3a_0^j \lambda_j, \end{bmatrix} \quad j = 3,4, \]  
\[ t_5 = \begin{bmatrix} 3a_0^j \lambda_j, \end{bmatrix} \quad j = 5. \]  

Solving equations (4.23)-(4.27) for A,B,C,D and E we obtain

\[ A = \frac{\Delta_1}{\Delta} \bar{g}(k,n), \quad B = \frac{\Delta_2}{\Delta} \bar{g}(k,n), \quad C = \frac{\Delta_3}{\Delta} \bar{g}(k,n). \]  

where

\[ \Delta = m_1(p_3m_5 - q_3m_4) - m_2(p_4m_5 - q_4m_4), \]  
\[ \Delta_1 = s_2(p_3m_1 - p_4m_2), \quad \Delta_2 = s_1(p_4m_2 - p_3m_1), \]  
\[ \Delta_3 = m_1m_4, \quad \Delta_4 = -m_2m_4, \quad \Delta_5 = m_3m_4, \]  
and

\[ m_1 = (r_5t_5 - r_5'^t_5), \quad m_2 = (r_2t_5 - r_5't_3), \quad m_3 = (r_3t_4 - r_4't_3), \]  
\[ m_4 = (s_1p_2 - s_2p_1), \quad m_5 = (s_1q_2 - s_2q_1). \]  

Using the Fourier inverse transformation (4.2) in (4.11) - (4.15) we obtain

\[ \phi = \frac{1}{2\pi} \int_{-\infty}^{\infty} (\text{Ai} e^{-\lambda_1^2z^2} + \text{Bi} e^{-\lambda_2^2z^2}) e^{-i(kx + nt)} dkdn, \]  
\[ \theta = \frac{1}{2\pi} \int_{-\infty}^{\infty} (a_1 \text{Ai} e^{-\lambda_1^2z^2} + a_2 \text{Bi} e^{-\lambda_2^2z^2}) e^{-i(kx + nt)} dkdn, \]  
\[ \psi = \frac{1}{2\pi} \int_{-\infty}^{\infty} (\text{Ce} e^{-\lambda_3^2z^2} + \text{De} e^{-\lambda_4^2z^2}) e^{-i(kx + nt)} dkdn, \]  
\[ \omega_z = \frac{1}{2\pi} \int_{-\infty}^{\infty} (a_3 \text{Ce} e^{-\lambda_3^2z^2} + a_4 \text{De} e^{-\lambda_4^2z^2}) e^{-i(kx + nt)} dkdn, \]  
\[ \phi = \frac{1}{2\pi} \int_{-\infty}^{\infty} \text{Ee} e^{-\lambda_5^2z^2} e^{-i(kx + nt)} dkdn. \]

Thus, using (4.40)-(4.44) we can obtain the displacement components, force stress, couple stress tensor, vector first moments, and the temperature field in the integral form

\[ u_1 = -\frac{1}{2\pi} \int_{-\infty}^{\infty} u_1^*(z,k,n) \bar{g}(k,n) e^{-i(kx + nt)} dkdn, \]  
\[ u_3 = -\frac{1}{2\pi} \int_{-\infty}^{\infty} u_3^*(z,k,n) \bar{g}(k,n) e^{-i(kx + nt)} dkdn, \]  
\[ \omega_z = \frac{1}{2\pi} \int_{-\infty}^{\infty} \omega_z^*(z,k,n) \bar{g}(k,n) e^{-i(kx + nt)} dkdn, \]
\[ \sigma_{33} = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-i(kx + nt)} \tilde{g}(k,n)e^{-i(kx + nt)} dk\,dn, \]  
(4.48)

\[ \sigma_{31} = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-i(kx + nt)} \tilde{g}(k,n)e^{-i(kx + nt)} dk\,dn, \]  
(4.49)

\[ \mu_{32} = -\frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-i(kx + nt)} \tilde{g}(k,n)e^{-i(kx + nt)} dk\,dn, \]  
(4.50)

\[ \beta_{3} = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-i(kx + nt)} \tilde{g}(k,n)e^{-i(kx + nt)} dk\,dn, \]  
(4.51)

\[ \theta = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-i(kx + nt)} \tilde{g}(k,n)e^{-i(kx + nt)} dk\,dn, \]  
(4.52)

where

\[ u_1^*(z, k, n) = \frac{1}{\Delta} \left[ ik (\Delta_1 e^{-\lambda_1 z} + \Delta_2 e^{-\lambda_2 z} + \lambda_3 \delta_3 e^{-\lambda_3 z} + \lambda_4 \delta_4 e^{-\lambda_4 z}) \right], \]  
(4.53)

\[ u_3^*(z, k, n) = \frac{1}{\Delta} \left[ \lambda_1 \Delta_1 e^{-\lambda_1 z} + \lambda_2 \Delta_2 e^{-\lambda_2 z} - ik (\Delta_3 e^{-\lambda_3 z} + \Delta_4 e^{-\lambda_4 z}) \right], \]  
(4.54)

\[ \omega_3^*(z, k, n) = \frac{1}{\Delta} \left[ a_3 \delta_3 e^{-\lambda_3 z} + a_4 \delta_4 e^{-\lambda_4 z} \right], \]  
(4.55)

\[ \sigma_{33}^*(z, k, n) = \frac{1}{\Delta} \left[ ((\lambda + 2\mu)\Delta_1^2 - k^2 \Delta_1 e^{-\lambda_1 z} \right. \]  
\[ + ((\lambda + 2\mu)\Delta_2^2 - k^2 \Delta_2 e^{-\lambda_2 z} \right) \]  
\[ - 2i \mu k (\lambda_3 \delta_3 e^{-\lambda_3 z} + \lambda_4 \delta_4 e^{-\lambda_4 z}) \right], \]  
(4.56)

\[ \sigma_{31}^*(z, k, n) = \frac{1}{\Delta} \left[ 2i \mu k (\lambda_1 \Delta_1 e^{-\lambda_1 z} + \lambda_2 \Delta_2 e^{-\lambda_2 z} \right. \]  
\[ + (\mu (k^2 + \lambda_3^2) + a_3 (\lambda_3^2 - k^2 - 2a_3)) \delta_3 e^{-\lambda_3 z} \]  
\[ + (\mu (k^2 + \lambda_4^2) + a_4 (\lambda_4^2 - k^2 - 2a_4)) \delta_4 e^{-\lambda_4 z} \right], \]  
(4.57)

\[ \mu_{32}^*(z, k, n) = \frac{1}{\Delta} \left[ (\gamma e) (\lambda_3 \delta_3 e^{-\lambda_3 z} + a_4 \delta_4 e^{-\lambda_4 z}) - i\beta_0 \kappa e^{-\lambda_5 z} \right], \]  
(4.58)

\[ \beta_3^*(z, k, n) = \frac{1}{\Delta} \left[ i\beta_0 k (\lambda_3 \delta_3 e^{-\lambda_3 z} + a_4 \delta_4 e^{-\lambda_4 z}) + 3\alpha_0 \lambda e^{-\lambda_5 z} \right], \]  
(4.59)

\[ \theta^*(z, k, n) = \frac{1}{\Delta} \left[ a_1 \Delta_1 e^{-\lambda_1 z} - a_2 \delta_2 e^{-\lambda_2 z} \right], \]  
(4.60)

5. PARTICULAR CASES:

(i) We consider a time periodic concentrated force acting at the origin in the direction of x-axis so that the loading function assumes the form

\[ g(x, t) = F_0(x) e^{-i\omega t}, \]  
(5.1)

where \( F \) is the magnitude of the force, \( \delta(x) \) is the Dirac function of distribution and \( \omega \) is the frequency.

The double Fourier transform of \( g(x, t) \) is
\[ \tilde{g}(k, \eta) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-\omega t} f(x) e^{-i\omega t} e^{i(kx+\eta t)} dx dt, \]
\[ = \frac{F}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{i(n-\omega)t} dt, \]
\[ = \sqrt{2\pi} F \delta(\eta-\omega). \] (5.2)

Thus from (4.45)-(4.60) with (5.2), we obtain
\[ u_1 = -\frac{F}{\sqrt{2\pi}} e^{-i\omega t} \int_{-\infty}^{\infty} [u_1^*(z,k,\eta)]_{n=\omega} e^{-ikx} dk, \] (5.3)
\[ u_3 = -\frac{F}{\sqrt{2\pi}} e^{-i\omega t} \int_{-\infty}^{\infty} [u_3^*(z,k,\eta)]_{n=\omega} e^{-ikx} dk, \] (5.4)
\[ \omega_2 = \frac{F}{\sqrt{2\pi}} e^{-i\omega t} \int_{-\infty}^{\infty} \omega_2^*(z,k,\eta)]_{n=\omega} e^{-ikx} dk, \] (5.5)
\[ \sigma_{33} = \frac{F}{\sqrt{2\pi}} e^{-i\omega t} \int_{-\infty}^{\infty} [\sigma_{33}^*(z,k,\eta)]_{n=\omega} e^{-ikx} dk, \] (5.6)
\[ \sigma_{31} = \frac{F}{\sqrt{2\pi}} e^{-i\omega t} \int_{-\infty}^{\infty} [\sigma_{31}^*(z,k,\eta)]_{n=\omega} e^{-ikx} dk, \] (5.7)
\[ u_{32} = \frac{F}{\sqrt{2\pi}} e^{-i\omega t} \int_{-\infty}^{\infty} [u_{32}^*(z,k,\eta)]_{n=\omega} e^{-ikx} dk, \] (5.8)
\[ \theta_3 = e^{-\frac{1}{3}} \frac{F}{\sqrt{2\pi}} e^{-i\omega t} \int_{-\infty}^{\infty} [\theta_3^*(z,k,\eta)]_{n=\omega} e^{-ikx} dk, \] (5.9)
\[ \theta = \frac{F}{\sqrt{2\pi}} e^{-i\omega t} \int_{-\infty}^{\infty} \theta^*(z,k,\eta)]_{n=\omega} e^{-ikx} dk. \] (5.10)

If we neglect the stretch effect, we recover the corresponding expressions for the displacements, stresses, and the temperature field in the form
\[ u_1 = -\frac{F}{\sqrt{2\pi}} e^{-i\omega t} \int_{-\infty}^{\infty} [M_1(z,k,\eta)]_{n=\omega} e^{-ikx} dk, \] (5.11)
\[ u_3 = -\frac{F}{\sqrt{2\pi}} e^{-i\omega t} \int_{-\infty}^{\infty} [M_2(z,k,\eta)]_{n=\omega} e^{-ikx} dk, \] (5.12)
\[ \omega_2 = \frac{F}{\sqrt{2\pi}} e^{-i\omega t} \int_{-\infty}^{\infty} [M_3(z,k,\eta)]_{n=\omega} e^{-ikx} dk, \] (5.13)
\[ \sigma_{33} = \frac{F}{\sqrt{2\pi}} e^{-i\omega t} \int_{-\infty}^{\infty} [M_3(z,k,\eta)]_{n=\omega} e^{-ikx} dk, \] (5.14)
\[ \sigma_{31} = \frac{F}{\sqrt{2\pi}} e^{-i\omega t} \int_{-\infty}^{\infty} [M_3(z,k,\eta)]_{n=\omega} e^{-ikx} dk, \] (5.15)
\[ \sigma_{32} = -\frac{F}{\sqrt{2\pi}} e^{-i\omega t} \int_{-\infty}^{\infty} [M_1(z,k,\eta)]_{n=\omega} e^{-ikx} dk, \] (5.16)
\[ \theta = \frac{F}{\sqrt{2\pi}} e^{-i\omega t} \int_{-\infty}^{\infty} [M_3(z,k,\eta)]_{n=\omega} e^{-ikx} dk, \] (5.17)
where

\[ M_1(z,k,n) = \frac{1}{\Delta} \left[ ik(\Delta_1^* e^{-\lambda_1 z} + \Delta_2^* e^{-\lambda_2 z} + \lambda_3^* e^{-\lambda_3 z} + \lambda_4^* e^{-\lambda_4 z}) \right], \tag{5.18} \]

\[ M_2(z,k,n) = \frac{1}{\Delta} \left[ \lambda_1^* e^{-\lambda_1 z} + \lambda_2^* e^{-\lambda_2 z} - ik \left( \Delta_3^* e^{-\lambda_3 z} + \Delta_4^* e^{-\lambda_4 z} \right) \right], \tag{5.19} \]

\[ M_3(z,k,n) = \frac{1}{\Delta} \left[ a_3^* e^{-\lambda_3 z} + a_4^* e^{-\lambda_4 z} \right], \tag{5.20} \]

\[ M_4(z,k,n) = \frac{1}{\Delta} \left[ (\lambda + 2\nu)^2 - k^2 - \nu a_1 \right] e^{-\lambda_2 z} \]

\[ + \left\{ (\lambda + 2\nu) e^{-\lambda_2 z} - k^2 - \nu a_2 \right\} \Delta_2^* e^{-\lambda_2 z} \]

\[ - 2iuk(\lambda_3^* e^{-\lambda_3 z} + \lambda_4^* e^{-\lambda_4 z}) \right], \tag{5.21} \]

\[ M_5(z,k,n) = \frac{1}{\Delta} \left[ 2iuk(\lambda_1^* e^{-\lambda_1 z} + \lambda_2^* e^{-\lambda_2 z}) \right] \]

\[ + \left\{ (\nu k^2 + \lambda_3^2) + a(\lambda_3^2 - k^2 - 2\alpha_3) \right\} \Delta_3^* e^{-\lambda_3 z} \]

\[ + \left\{ (\nu k^2 + \lambda_4^2) + a(\lambda_4^2 - k^2 - 2\alpha_4) \right\} \Delta_4^* e^{-\lambda_4 z} \right], \tag{5.22} \]

\[ M_6(z,k,n) = \frac{1}{\Delta} \left[ (\gamma + \epsilon)(\lambda_3 a_3^* e^{-\lambda_3 z} + \lambda_4 a_4^* e^{-\lambda_4 z}) \right], \tag{5.23} \]

\[ M_7(z,k,n) = \frac{1}{\Delta} \left[ a_1^* e^{-\lambda_1 z} + a_2^* e^{-\lambda_2 z} \right], \tag{5.24} \]

and

\[ \delta^* = (p_3 r_4 - p_4 r_3)(s_2 q_1 - s_1 q_2) - (q_3 r_4 - q_4 r_3)(s_2 p_1 - s_1 p_2), \tag{5.25} \]

\[ \delta_1^* = s_2 (p_3 r_4 - p_4 r_3), \tag{5.26} \]

\[ \delta_2^* = s_1 (p_3 r_4 - p_4 r_3), \tag{5.27} \]

\[ \delta_3^* = r_4 (s_1 p_2 - s_2 p_1), \tag{5.28} \]

\[ \delta_4^* = r_3 (s_1 p_2 - s_2 p_1). \tag{5.29} \]

These results agree with those obtained by Acharya and Sengupta [8].

(ii) In this case we consider a torque with its axis parallel to the z-axis so that \( g(x,t) \) can be written as

\[ g(x,t) = G[\delta(x-a) - \delta(x+a)] e^{-i\omega t} \tag{5.30} \]

where \( G \) is the magnitude of the force.

The double Fourier transformation of (5.30) gives

\[ \tilde{g}(k,n) = \frac{G}{2\pi} e^{-\omega t} \int_{-\infty}^{\infty} [\delta(x-a) - \delta(x+a)] e^{i(kx+\eta t)} e^{-i\omega t} dx dt, \]

\[ = 2i \sqrt{(2\pi)} G \sin (ka) \delta(\eta - \omega). \tag{5.31} \]
Then from (4.45) - (4.60) with (5.31), we obtain

\[ u_1 = -iG \sqrt{(2/\pi)}e^{-i\omega t} \sum_{\omega} \left[ u_1^*(z,k,n) \right]_{\eta=\omega} \sin(ka)e^{-ikx}dk, \] (5.32)

\[ u_3 = -iG \sqrt{(2/\pi)}e^{-i\omega t} \sum_{\omega} \left[ u_3^*(z,k,n) \right]_{\eta=\omega} \sin(ka)e^{-ikx}dk, \] (5.33)

\[ \omega_2 = iG \sqrt{(2/\pi)}e^{-i\omega t} \sum_{\omega} \left[ \omega_2^*(z,k,n) \right]_{\eta=\omega} \sin(ka)e^{-ikx}dk, \] (5.34)

\[ \sigma_{33} = iG \sqrt{(2/\pi)}e^{-i\omega t} \sum_{\omega} \left[ \sigma_{33}^*(z,k,n) \right]_{\eta=\omega} \sin(ka)e^{-ikx}dk, \] (5.35)

\[ \sigma_{31} = iG \sqrt{(2/\pi)}e^{-i\omega t} \sum_{\omega} \left[ \sigma_{31}^*(z,k,n) \right]_{\eta=\omega} \sin(ka)e^{-ikx}dk, \] (5.36)

\[ \nu_{32} = -iG \sqrt{(2/\pi)}e^{-i\omega t} \sum_{\omega} \left[ \nu_{32}^*(z,k,n) \right]_{\eta=\omega} \sin(ka)e^{-ikx}dk, \] (5.37)

\[ \theta_3 = -\frac{iG}{3} \sqrt{(2/\pi)}e^{-i\omega t} \sum_{\omega} \left[ \theta_3^*(z,k,n) \right]_{\eta=\omega} \sin(ka)e^{-ikx}dk, \] (5.38)

\[ \theta = iG \sqrt{(2/\pi)}e^{-i\omega t} \sum_{\omega} \left[ \theta(z,k,n) \right]_{\eta=\omega} \sin(ka)e^{-ikx}dk. \] (5.39)

In the absence of the stretch effect, we obtain the corresponding expressions for the displacements, stresses and the temperature field in the form

\[ u_1 = -iG \sqrt{(2/\pi)}e^{-i\omega t} \sum_{\omega} \left[ M_1(z,k,n) \right]_{\eta=\omega} \sin(ka)e^{-ikx}dk, \] (5.40)

\[ u_3 = -iG \sqrt{(2/\pi)}e^{-i\omega t} \sum_{\omega} \left[ M_3(z,k,n) \right]_{\eta=\omega} \sin(ka)e^{-ikx}dk, \] (5.41)

\[ \omega_2 = iG \sqrt{(2/\pi)}e^{-i\omega t} \sum_{\omega} \left[ M_3(z,k,n) \right]_{\eta=\omega} \sin(ka)e^{-ikx}dk, \] (5.42)

\[ \sigma_{33} = iG \sqrt{(2/\pi)}e^{-i\omega t} \sum_{\omega} \left[ M_4(z,k,n) \right]_{\eta=\omega} \sin(ka)e^{-ikx}dk, \] (5.43)

\[ \sigma_{31} = iG \sqrt{(2/\pi)}e^{-i\omega t} \sum_{\omega} \left[ M_5(z,k,n) \right]_{\eta=\omega} \sin(ka)e^{-ikx}dk, \] (5.44)

\[ \nu_{32} = -iG \sqrt{(2/\pi)}e^{-i\omega t} \sum_{\omega} \left[ M_6(z,k,n) \right]_{\eta=\omega} \sin(ka)e^{-ikx}dk, \] (5.45)

\[ \theta = iG \sqrt{(2/\pi)}e^{-i\omega t} \sum_{\omega} \left[ M_7(z,k,n) \right]_{\eta=\omega} \sin(ka)e^{-ikx}dk. \] (5.46)

These results also agree with the corresponding results without stretch.

6. **CONCLUSION.**

The displacement field, force stress, couple stress, temperature field and vector first moment have been obtained. It is noted that the displacement field, force stress, couple stress and temperature field involve the parameters \( a_0, \beta_0 \) and \( \eta_0 \) of the micropolar elastic media with stretch. In addition to the displacements, force stress, couple stress, and temperature field, vector first moment \( \beta_j \) has been determined which vanishes in the case of thermo-micropolar elasticity. Some numerical calculation for specific models of physical interest will be carried out and will be communicated in a subsequent paper.
Acknowledgement: The second author expresses his grateful thanks to Guru Nanak Dev University, Amritsar for providing financial assistance during the preparation of the paper.

REFERENCES


Special Issue on
Time-Dependent Billiards

Call for Papers

This subject has been extensively studied in the past years for one-, two-, and three-dimensional space. Additionally, such dynamical systems can exhibit a very important and still unexplained phenomenon, called as the Fermi acceleration phenomenon. Basically, the phenomenon of Fermi acceleration (FA) is a process in which a classical particle can acquire unbounded energy from collisions with a heavy moving wall. This phenomenon was originally proposed by Enrico Fermi in 1949 as a possible explanation of the origin of the large energies of the cosmic particles. His original model was then modified and considered under different approaches and using many versions. Moreover, applications of FA have been of a large broad interest in many different fields of science including plasma physics, astrophysics, atomic physics, optics, and time-dependent billiard problems and they are useful for controlling chaos in Engineering and dynamical systems exhibiting chaos (both conservative and dissipative chaos).

We intend to publish in this special issue papers reporting research on time-dependent billiards. The topic includes both conservative and dissipative dynamics. Papers discussing dynamical properties, statistical and mathematical results, stability investigation of the phase space structure, the phenomenon of Fermi acceleration, conditions for having suppression of Fermi acceleration, and computational and numerical methods for exploring these structures and applications are welcome.

To be acceptable for publication in the special issue of Mathematical Problems in Engineering, papers must make significant, original, and correct contributions to one or more of the topics above mentioned. Mathematical papers regarding the topics above are also welcome.

Authors should follow the Mathematical Problems in Engineering manuscript format described at http://www.hindawi.com/journals/mpe/. Prospective authors should submit an electronic copy of their complete manuscript through the journal Manuscript Tracking System at http://mts.hindawi.com/ according to the following timetable:

<table>
<thead>
<tr>
<th>Event</th>
<th>Date</th>
</tr>
</thead>
<tbody>
<tr>
<td>Manuscript Due</td>
<td>March 1, 2009</td>
</tr>
<tr>
<td>First Round of Reviews</td>
<td>June 1, 2009</td>
</tr>
<tr>
<td>Publication Date</td>
<td>September 1, 2009</td>
</tr>
</tbody>
</table>

Guest Editors

Edson Denis Leonel, Department of Statistics, Applied Mathematics and Computing, Institute of Geosciences and Exact Sciences, State University of São Paulo at Rio Claro, Avenida 24A, 1515 Bela Vista, 13506-700 Rio Claro, SP, Brazil; edleonel@rc.unesp.br

Alexander Loskutov, Physics Faculty, Moscow State University, Vorob'evy Gory, Moscow 119992, Russia; loskutov@chaos.phys.msu.ru