From the enumerative generating function of an abstract adjacency statistic, we deduce
the mean and variance of the variation on random permutations, rearrangements, com-
positions, and bounded integer sequences of finite length.

1. Introduction

When the finite sequence of integers \( w = 1,3,2,2,4,3 \) is sketched as below,

\[
w = \begin{array}{c}
1 \\
2 \\
2 \\
3 \\
3 \\
4 \\
\end{array}
\]

its most compelling aspect is its vertical variation, that is, the sum of the vertical distances
between its adjacent terms. Denoted by \( \text{var} \, w \), the vertical variation of the sequence in
(1.1) is \( \text{var} \, w = 2 + 1 + 0 + 2 + 1 = 6 \). Our purpose here is to compute the mean and vari-
ance of \( \text{var} \) on four classical sets of combinatorial sequences.

To formalize matters and place our problem in the context of other work, let \([m]^n\)
denote the set of sequences \( w = x_1x_2 \cdots x_n \) of length \( n \) with each \( x_i \in \{1,2,\ldots,m\} \). For
a real-valued function \( f \) on \([m]^2\), the \( f \)-adjacency number of \( w = x_1x_2 \cdots x_n \in [m]^n \) is
defined to be

\[
\text{adf} \, w = \sum_{k=1}^{n-1} f(x_kx_{k+1}).
\]
Some specializations of the $f$-adjacency number have been considered elsewhere. For instance, if $f(xy)$ is 1 when $x < y$ and 0 otherwise, then $adf w$ is known as the *rise number* of $w$ [1, 3, 4]. For the selection $f(xy) = |y - x|$, $adf w = var w$. In a sorting problem of computer science, Levcopoulos and Petersson [5] introduced the related notion of oscillation for $adf$ over $\{1, 2, \ldots, n\}$ as a measure of the presortedness of a sequence of $n$ distinct numbers. In [6], compositions were enumerated by their *ascent variation*, the $f$-adjacency statistic induced by $f(xy) = y - x$ if $x < y$ and 0 otherwise. For the case $f(xy) = h(|y - x|)$ where $h$ is a linear, convex, or concave increasing real-valued function, Chao and Liang [2] described the arrangements of $n$ distinct integers for which $adf$ achieves its extreme values.

Besides considering the distribution of $var$ on the set $[m]^n$, we also consider it on the set of rearrangements $R_n(i_1, i_2, \ldots, i_m)$ consisting of sequences of length $n = i_1 + i_2 + \cdots + i_m$ which contain $l$ exactly $i_l$ times, on the set of permutations $S_n = R_n(1, 1, \ldots, 1)$ of $\{1, 2, \ldots, n\}$, and on the set of compositions of $m$ into $n$ parts $C_n(m) = \{x_1 x_2 \cdots x_n \in [m]^n : x_1 + x_2 + \cdots + x_n = m\}$. For $m, n \geq 2$, Table 1.1 displays the mean and variance of $var$ on these four sets. The $k$th falling factorial of $n$ is $n^k = n(n - 1) \cdots (n - k + 1)$, $i = (i_1, i_2, \ldots, i_m)$, and, for $r$ a real number, $\lfloor r \rfloor$ denotes the greatest integer less than or equal to $r$. The results in Table 1.1 are new. David and Barton [3, Chapter 10] present the distributions of several statistics (some $f$-adjacency numbers, some not) primarily on permutations. We also note that Tiefenbruck [7] derived a generating function for compositions with bounded parts by a close relative of $var$. We leave open questions concerning the asymptotic behavior of $var$.

### 2. Enumerative factorial moments for $f$-adjectives

Before working specifically with $var$, we discuss the enumerative generating function for $adf$ on sequences as developed by Fédou and Rawlings [4]. Let $[m]^*$ denote the set of sequences of $1, 2, \ldots, m$ of finite length (including the empty sequence of length 0). For $w = x_1 x_2 \cdots x_n \in [m]^*$, we define $Z^w = z_{x_1} z_{x_2} \cdots z_{x_n}$. The enumerative generating function for $adf$ over $[m]^*$ is then defined to be $G(p) = \sum_{w \in [m]^*} p^{\text{adf}_w} Z^w$.

By manipulating $G(p)$, we will obtain all of the information in Table 1.1 (and more). As a brief outline of our approach, note that the coefficient of $p^k z_1^{i_1} z_2^{i_2} \cdots z_m^{i_m}$ in $G(p)$ is

<table>
<thead>
<tr>
<th>Sequences</th>
<th>Expected value of $var$</th>
<th>Variance of $var$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_n$</td>
<td>$n^2 - 1$</td>
<td>$(n-2)(n+1)(4n-7)$</td>
</tr>
<tr>
<td>$[m]^n$</td>
<td>$(n-1)(m^2-1)$</td>
<td>$(m^2-1)(6m^2n+6n-7m^2-2)$</td>
</tr>
<tr>
<td>$R_n(i)$</td>
<td>$\frac{2}{n} \sum_{1 \leq x &lt; y \leq m} (y-x) i_x i_y$</td>
<td>See (3.10)</td>
</tr>
<tr>
<td>$C_n(m)$</td>
<td>$\frac{2(n-1)}{(m-1)^{a-1}} \sum_{x=1}^{\lfloor m/2 \rfloor} (m-2x)^{a-1}$</td>
<td>See (3.18)</td>
</tr>
</tbody>
</table>

Table 1.1
just the number of rearrangements \( w \) in \( R_n(i) \) with \( \text{adf} w = k \). Thus, by dividing the coefficient of \( z_1^i z_2^i \cdots z_m^i \) in \( G'(1) \) by the cardinality of \( R_n(i) \), we will obtain the mean of \( \text{adf} \). So, in general, we compute the \( d \)th enumerative factorial moment \( G^{(d)}(1) = \sum_{w \in [m]^*} (\text{adf} w)^d \quad Z_w \).

From the work of Fédou and Rawlings [4], it follows that

\[
G(p) = \frac{1}{D(p)},
\]

where

\[
D(p) = 1 - \sum_{n \geq 1} \sum_{x_1 \cdots x_n \in [m]^n} Z_{x_1 \cdots x_n} \prod_{k=1}^{n-1} (p^f(x_k, x_{k+1}) - 1).
\]

Examples are presented in [4, 6] for which \( D \) has a closed form. We do not know a closed form for \( D \) when \( \text{adf} = \text{var} \) (that is, when \( f(x, y) = |y - x| \)). Nevertheless, (2.1) is still useful in computing the mean and variance of \( \text{var} \).

Although the formula for taking the \( d \)-fold derivative with respect to \( p \) of a function of the form in (2.1) is known, we provide a short derivation. To avoid the quotient and chain rules, rewrite (2.1) as \( GD = 1 \). Differentiating the latter \( d \) times, \( d \geq 1 \), and dividing by \( d! \) gives

\[
\sum_{j=0}^{d} \frac{G^{(d-j)} D^{(j)}}{(d-j)! j!} = 0.
\]

To solve for \( G^{(d)} \), consider the system

\[
\begin{align*}
\frac{G^{(d)}}{d!} & \frac{D^{(0)}}{0!} + \frac{G^{(d-1)}}{(d-1)!} \frac{D^{(1)}}{1!} + \frac{G^{(d-2)}}{(d-2)!} \frac{D^{(2)}}{2!} + \cdots + \frac{G^{(0)}}{0!} \frac{D^{(d)}}{d!} = 0, \\
\frac{G^{(d-1)}}{(d-1)!} & \frac{D^{(0)}}{0!} + \frac{G^{(d-2)}}{(d-2)!} \frac{D^{(1)}}{1!} + \cdots + \frac{G^{(0)}}{0!} \frac{D^{(d-1)}}{(d-1)!} = 0, \\
& \vdots \\
\frac{G^{(1)}}{1!} & \frac{D^{(0)}}{0!} + \frac{G^{(0)}}{0!} \frac{D^{(1)}}{1!} = 0,
\end{align*}
\]

where the top \( d \) equations arise from repeated application of (2.3). Cramer’s rule applied
to the above system yields

$$G^{(d)}(d) = \left(\begin{array}{cccc} \frac{D^{(1)}}{1!} & \frac{D^{(2)}}{2!} & \frac{D^{(3)}}{3!} & \cdots & \frac{D^{(d)}}{d!} \\ \frac{D^{(0)}}{0!} & \frac{D^{(1)}}{2!} & \frac{D^{(2)}}{3!} & \cdots & \frac{D^{(d-1)}}{(d-1)!} \\ \frac{D^{(0)}}{0!} & \frac{D^{(1)}}{1!} & \frac{D^{(2)}}{2!} & \cdots & \frac{D^{(d-2)}}{(d-2)!} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \frac{D^{(0)}}{0!} & \frac{D^{(1)}}{1!} \end{array}\right) \tag{2.5}$$

which, when expanded, implies that

$$G^{(d)} = \sum_{\nu=1}^{d} \left(\frac{1}{D^{\nu+1}} \sum_{j_1\cdots j_\nu = d} \left(\begin{array}{c} d \\ j_1 \cdots j_\nu \end{array}\right) D^{(j_1)} \cdots D^{(j_\nu)}\right). \tag{2.6}$$

To determine the enumerative factorial moment $G^{(d)}(1)$, we see from (2.2) that

$$D^{(j)}(1) = -\sum_{r=2}^{j+1} D^{(r)} \tag{2.7}$$

where

$$D^{(j)} = \sum_{x_1,\ldots,x_r \in [m]^j} Z^{x_1 \cdots x_r} \sum_{l_1 \cdots l_{r-1} = j} \left(\begin{array}{c} j \\ l_1 \cdots l_{r-1} \end{array}\right) \prod_{k=1}^{r-1} f(x_k x_{k+1})^2. \tag{2.8}$$

For instance,

$$D_2' = \sum_{x,y \in [m]^2} f(xy)z_x z_y, \quad D_2'' = \sum_{x,y \in [m]^2} f(xy)^2 z_x z_y, \quad D_3''' = 2 \sum_{vx \in [m]^3} f(vx) f(xy) z_x z_y. \tag{2.9}$$

Further setting $\vec{j} = (j_1,\ldots,j_\nu)$, $s(\vec{j}) = j_1 + \cdots + j_\nu$,

$$\left(\begin{array}{c} d \\ j \end{array}\right) = \left(\begin{array}{c} d \\ j_1 \cdots j_\nu \end{array}\right), \quad \text{and} \quad D^{(\vec{j})}_\mu = \sum_{r_1,\ldots,r_\nu = \mu} D^{(j_1)}_1 \cdots D^{(j_\nu)}_r. \tag{2.10}$$

it follows from (2.6) and (2.7) that

$$G^{(d)}(1) = \sum_{\nu=1}^{d} \frac{1}{D^{\nu+1}(1)} \sum_{s(\vec{j}) = d} \left(\begin{array}{c} d \\ j \end{array}\right) \sum_{\mu=2^\nu} D^{(\vec{j})}_\mu. \tag{2.11}$$
As $D(1) = 1 - (z_1 + \cdots + z_m)$, extracting the contributions made by all $w \in [m]^n$ from both sides of (2.11) gives the $d$th enumerative factorial moment of $\text{adf}$ over $[m]^n$ as

$$
\sum_{w \in [m]^n} (\text{adf } w)^d Z^w = \sum_{y=1}^{d} \sum_{s(j)=d \atop j_k \geq 1} \binom{d}{j} \sum_{\mu=2y}^{n+y-\mu} \left( \sum_{i=1}^{m} z_i \right)^{n-\mu} D^{(j)}_{\eta} (2.12)
$$

valid for $d \geq 1$. When $d = 1, 2$, (2.9) and (2.12) imply that

$$
\sum_{w \in [m]^n} \text{adf } w Z^w = (n-1) \left( \sum_{i=1}^{m} z_i \right)^{n-2} \sum_{xy \in [m]^2} f(xy) z_x z_y \quad (2.13)
$$

and that

$$
\sum_{w \in [m]^n} (\text{adf } w)^2 Z^w = (n-1) \left( \sum_{i=1}^{m} z_i \right)^{n-2} \sum_{xy \in [m]^2} (f(xy))^2 z_x z_y \\
+ 2(n-2) \left( \sum_{i=1}^{m} z_i \right)^{n-3} \sum_{vxz \in [m]^3} f(vx) f(xy) z_v z_x z_y \\
+ (n-2)(n-3) \left( \sum_{i=1}^{m} z_i \right)^{n-4} \left( \sum_{xy \in [m]^2} f(xy) z_x z_y \right)^2. \quad (2.14)
$$

3. Discussion of Table 1.1

The entries in Table 1.1 are consequences of (2.13) and (2.14) with $f(xy) = |y-x|$ and with appropriate substitutions for $Z$. For the mean and variance of $\text{var}$ on the set of bounded sequences $[m]^n$, put $z_i = 1$ for $1 \leq i \leq m$. Noting that

$$
\sum_{xy \in [m]^2} |y-x| = \sum_{1 \leq x \leq y \leq m} 2(y-x) = 2 \binom{m+1}{3}, \quad (3.1)
$$

it follows from (2.13) that the mean of $\text{var}$ on $[m]^n$ is

$$
\frac{1}{m^n} \sum_{w \in [m]^n} \text{var } w = 2(n-1)m^{n-2} \binom{m+1}{3} = \frac{(n-1)(m^2 - 1)}{3m}. \quad (3.2)
$$

As

$$
\sum_{xy \in [m]^2} |y-x|^2 = 4 \binom{m+1}{4}. \quad (3.3)
$$
and as

\[
\sum_{vxy \in [m]^n} |x - v||y - x| = \sum_{1 \leq v < x \leq y \leq m} 2(x - v)(y - x)
\]

\[
+ \sum_{1 \leq x < y \leq v \leq m} 4(v - x)(y - x) - \sum_{1 \leq x < y \leq m} 2(y - x)^2 \tag{3.4}
\]

\[
= \frac{7m^2 - 8}{10} \binom{m + 1}{3},
\]

(2.14) implies that

\[
\frac{1}{mn} \sum_{w \in [m]^n} (\text{var} w)^2 = \frac{4(n - 1)}{m^2} \left( \frac{m + 1}{4} \right) + \frac{(n - 2)(7m^2 - 8)}{5m^3} \left( \frac{m + 1}{3} \right)
\]

\[
+ \frac{4(n - 2)(n - 3)}{m^4} \left( \frac{m + 1}{3} \right)^2. \tag{3.5}
\]

Then, subbing the last result into

\[
\frac{1}{mn} \sum_{w \in [m]^n} (\text{var} w)^2 + \frac{(n - 1)(m^2 - 1)}{3m} - \left( \frac{(n - 1)(m^2 - 1)}{3m} \right)^2 \tag{3.6}
\]

and simplifying gives the variance of var as recorded in Table 1.1.

For \(R_n(\vec{i})\), extracting the coefficient of \(z_1^{i_1}z_2^{i_2} \ldots z_m^{i_m}\) from (2.13) leads to

\[
\sum_{w \in R_n(\vec{i})} \text{var} w = 2(n - 1) \sum_{1 \leq x < y \leq m} (y - x) \binom{n}{i_1 \ldots i_x - 1 \ldots i_y - 1 \ldots i_m} \tag{3.7}
\]

As the cardinality of \(R_n(\vec{i})\) is

\[
\binom{n}{i_1 i_2 \ldots i_m} = \binom{n}{\vec{i}}, \tag{3.8}
\]

it follows that the mean of var on \(R_n(\vec{i})\) is

\[
\binom{n}{\vec{i}}^{-1} \sum_{w \in R_n(\vec{i})} \text{var} w = \frac{2}{n} \sum_{1 \leq x < y \leq m} (y - x)i_x i_y. \tag{3.9}
\]

Let \(\vec{i}_r = (i_1, \ldots, i_r - 1, \ldots, i_n)\). For example, \((3,2,1,4)_{3\{1,2\}3} = (3,1,-1,4)\). The variance on \(R_n(\vec{i})\) is then

\[
\binom{n}{\vec{i}}^{-1} \sum_{w \in R_n(\vec{i})} \text{var} w^2 + \frac{2}{n} \sum_{1 \leq x < y \leq m} (y - x)i_x i_y - \left( \frac{2}{n} \sum_{1 \leq x < y \leq m} (y - x)i_x i_y \right)^2. \tag{3.10}
\]
where, upon extraction of the coefficient of $z_1^i z_2^j \ldots z_m^i$ from (2.14), we have

\[
\sum_{w \in R_n(i)} (\text{var} w)^2 = (n - 1)! \sum_{1 \leq x, y \leq m} |y - x|^2 \binom{n - 2}{i(x,y)} \\
+ 2(n - 2)! \sum_{1 \leq v, x, y \leq m} |x - v||y - x| \binom{n - 3}{i(v,x,y)} \\
+ (n - 2)(n - 3) \sum_{1 \leq u, v, x, y \leq m} |v - u||y - x| \binom{n - 4}{i(u,v,x,y)}. (3.11)
\]

The permutation entries in Table 1.1 follow from (3.9) and (3.10). Selecting $m = n$ and $i_k = 1$ for $1 \leq k \leq n$ in (3.9) reveals the mean of var on $S_n$ as

\[
\frac{1}{n!} \sum_{w \in S_n} \text{var} w = \frac{2}{n} \sum_{1 \leq x, y \leq n} (y - x) = \frac{2}{n} \binom{n + 1}{3} = \frac{n^2 - 1}{3}. (3.12)
\]

From (3.11), with $m = n$ and $i_k = 1$ for $1 \leq k \leq n$,

\[
\sum_{w \in S_n} (\text{var} w)^2 = (n - 1)! \sum_{1 \leq x, y \leq n} |y - x|^2 \\
+ 2(n - 2)! \sum_{1 \leq v, x, y \leq n} |x - v||y - x| \\
+ (n - 2)! \sum_{1 \leq u, v, x, y \leq m} |v - u||y - x| \\
= \left(\frac{4}{15}\right) (n - 2)! (10n^2 + 14n - 27) \binom{n + 1}{4}. (3.13)
\]

So the variance of var on $S_n$ is

\[
\frac{1}{n!} \sum_{w \in S_n} \text{var} w^2 + \frac{n^2 - 1}{3} - \left(\frac{n^2 - 1}{3}\right)^2 = \frac{(n - 2)(n + 1)(4n - 7)}{90}. (3.14)
\]

For $w = x_1 \ldots x_n \in [m]^n$, let $\|w\| = x_1 + \cdots + x_n$. For the composition results in Table 1.1, set $z_k = q^k$ for $1 \leq k \leq m$. Then (2.13) implies that

\[
\sum_{w \in [m]^n} \text{var} w q^{\|w\|} = (n - 1)q^{n-2} \left(\frac{1 - q^m}{1 - q}\right)^{n-2} \sum_{1 \leq x, y \leq m} |y - x|q^{x+y}. (3.15)
\]
and (2.14) leads to

\[
\sum_{w \in [m]^n} (\text{var } w)^2 \|w\| = (n - 1) q^{n-2} \left( \frac{1 - q^m}{1 - q} \right)^{n-2} \sum_{1 \leq x, y \leq m} |y - x|^2 q^{x+y} \\
+ 2(n - 2) q^{n-3} \left( \frac{1 - q^m}{1 - q} \right)^{n-3} \sum_{1 \leq v, x, y \leq m} |x - v||y - x| q^{v+x+y} \\
+ (n - 2)(n - 3) q^{n-4} \left( \frac{1 - q^m}{1 - q} \right)^{n-4} \sum_{1 \leq u, v, x, y \leq m} |v - u||y - x| q'^{u+v+x+y}. 
\]

(3.16)

Extracting the coefficient of \( q^m \) from (3.15) to obtain

\[
\sum_{w \in \mathcal{C}_n(m)} \text{var } w = 2(n - 1) \sum_{1 \leq x < y \leq m} (y - x) \left( \frac{m - 1 - x - y}{n - 3} \right) \\
= 2(n - 1) \sum_{1 \leq x \leq \lfloor m/2 \rfloor} \left( \frac{m - 2x}{n - 1} \right) 
\]

(3.17)

and then dividing by the cardinality \( \binom{m-1}{n-1} \) of \( \mathcal{C}_n(m) \) gives the mean of var as stated in Table 1.1. The variance is

\[
\left( \frac{m - 1}{n - 1} \right)^{-1} \sum_{w \in \mathcal{C}_n(m)} \text{var } w \|w\| + \frac{2(n - 1)}{(m - 1)^{\frac{n-1}{2}}} \sum_{1 \leq x \leq \lfloor m/2 \rfloor} (m - 2x)^{\frac{n-1}{2}} \\
- \left( \frac{2(n - 1)}{(m - 1)^{\frac{n-1}{2}}} \sum_{1 \leq x \leq \lfloor m/2 \rfloor} (m - 2x)^{\frac{n-1}{2}} \right)^2, 
\]

(3.18)

where, pulling the coefficient of \( q^m \) from (3.16), we have

\[
\sum_{w \in \mathcal{C}_n(m)} (\text{var } w)^2 = (n - 1) \sum_{1 \leq x, y \leq m} |y - x|^2 \left( \frac{m - 1 - x - y}{n - 3} \right) \\
+ 2(n - 2) \sum_{1 \leq v, x, y \leq m} |x - v||y - x| \left( \frac{m - 1 - v - x - y}{n - 4} \right) \\
+ (n - 2)(n - 3) \sum_{1 \leq u, v, x, y \leq m} |v - u||y - x| \left( \frac{m - 1 - u - v - x - y}{n - 5} \right). 
\]

(3.19)

The sums in (3.19) are marginally simplified. For instance,

\[
\sum_{1 \leq x, y \leq m} |y - x|^2 \left( \frac{m - 1 - x - y}{n - 3} \right) = 4 \sum_{1 \leq x \leq \lfloor m/2 \rfloor} \left( \frac{m - 2x}{n} \right). 
\]

(3.20)
As a part of the second sum on the right-hand side of (3.19), we note that

\[
\sum_{1 \leq v < x < y \leq m} (x - v)(y - x) \binom{m - 1 - v - x - y}{n - 4}
\]

\[
= \sum_{2 \leq x \leq \lfloor (m+1)/2 \rfloor} \left( \binom{m - 3x + 1}{n} - \binom{m - 2x + 1}{n} \right) + x \binom{m - 2x}{n - 1}.
\]

(3.21)

The four-fold sums arising in the last sum in (3.19) reduce to double sums.

Acknowledgment

This work is based on work supported by the National Science Foundation under Grant no. 0097392.

References


Rudolfo Angeles: Department of Mathematics, College of Science and Mathematics, California Polytechnic State University, San Luis Obispo, CA 93407, USA

Current address: Department of Statistics, Stanford University, Palo Alto, CA 94305-4065, USA
E-mail address: rangeles@stanford.edu

Don Rawlings: Department of Mathematics, College of Science and Mathematics, California Polytechnic State University, San Luis Obispo, CA 93407, USA

E-mail address: drawling@calpoly.edu

Lawrence Sze: Department of Mathematics, College of Science and Mathematics, California Polytechnic State University, San Luis Obispo, CA 93407, USA

E-mail address: lsze@calpoly.edu

Mark Tiefenbruck: Department of Mathematics, College of Science and Mathematics, California Polytechnic State University, San Luis Obispo, CA 93407, USA

Current address: 8989 Jasmine Lane Street, Cottage Grove, MN 55016-3436, USA
E-mail address: mdash@alumni.northwestern.edu
Special Issue on
Boundary Value Problems on Time Scales

Call for Papers

The study of dynamic equations on a time scale goes back to its founder Stefan Hilger (1988), and is a new area of still fairly theoretical exploration in mathematics. Motivating the subject is the notion that dynamic equations on time scales can build bridges between continuous and discrete mathematics; moreover, it often reveals the reasons for the discrepancies between two theories.

In recent years, the study of dynamic equations has led to several important applications, for example, in the study of insect population models, neural network, heat transfer, and epidemic models. This special issue will contain new researches and survey articles on Boundary Value Problems on Time Scales. In particular, it will focus on the following topics:

- Existence, uniqueness, and multiplicity of solutions
- Comparison principles
- Variational methods
- Mathematical models
- Biological and medical applications
- Numerical and simulation applications

Before submission authors should carefully read over the journal’s Author Guidelines, which are located at http://www.hindawi.com/journals/ade/guidelines.html. Authors should follow the Advances in Difference Equations manuscript format described at the journal site http://www.hindawi.com/journals/ade/. Articles published in this Special Issue shall be subject to a reduced Article Processing Charge of $200 per article. Prospective authors should submit an electronic copy of their complete manuscript through the journal Manuscript Tracking System at http://mts.hindawi.com/ according to the following timetable:

<table>
<thead>
<tr>
<th>Event</th>
<th>Date</th>
</tr>
</thead>
<tbody>
<tr>
<td>Manuscript Due</td>
<td>April 1, 2009</td>
</tr>
<tr>
<td>First Round of Reviews</td>
<td>July 1, 2009</td>
</tr>
<tr>
<td>Publication Date</td>
<td>October 1, 2009</td>
</tr>
</tbody>
</table>

Lead Guest Editor

Alberto Cabada, Departamento de Análise Matemática, Universidade de Santiago de Compostela, 15782 Santiago de Compostela, Spain; alberto.cabada@usc.es

Guest Editor

Victoria Otero-Espinar, Departamento de Análise Matemática, Universidade de Santiago de Compostela, 15782 Santiago de Compostela, Spain; mvictoria.otero@usc.es