We prove that if $A$ is a $C$-algebra, then for each $a \in A$, $A_a = \{x \in A/ x \leq a\}$ is itself a $C$-algebra and is isomorphic to the quotient algebra $A/\theta_a$ of $A$ where $\theta_a = \{(x,y) \in A \times A/ x = a \land y\}$. If $A$ is a $C$-algebra with $T$, we prove that for every $a \in B(A)$, the centre of $A$, $A$ is isomorphic to $A_a \times A_{a'}$ and that if $A$ is isomorphic $A_1 \times A_2$, then there exists $a \in B(A)$ such that $A_1$ is isomorphic $A_a$ and $A_2$ is isomorphic to $A_{a'}$. Using this decomposition theorem, we prove that if $a, b \in B(A)$ with $a \land b = F$, then $A_a$ is isomorphic to $A_b$ if and only if there exists an isomorphism $\phi$ on $A$ such that $\phi(a) = b$.

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Introduction

In [1], Guzmán and Squier introduced the variety of $C$-algebras as a class of algebras of type $(2,2,1)$ satisfying certain identities and proved that this variety is generated by the 3-element algebra $C = \{T,F,U\}$ which is the algebraic semantic of the three valued conditional logic. In [3] Swamy et al. introduced the concept of the centre $B(A) = \{x \in A/ x \lor x' = T\}$ of a $C$-algebra $A$ with $T$ and proved that $B(A)$ is a Boolean algebra with induced operations and is equivalent to the Boolean Centre of $A$. In [2], Rao and Sundarayya defined a partial ordering on a $C$-algebra $A$ and the properties of $A$ as a poset are studied.

In this paper, we prove that if $A$ is a $C$-algebra, then for each $x \in A$, $A_x = \{s \in A/ s \leq x\}$ is itself a $C$-algebra and is isomorphic to the quotient algebra $A/\theta_x$, where $\theta_x = \{(s,t) \in A \times A/ x \land s = x \land t\}$. If $A$ is a $C$-algebra with $T$ then, for every $a \in B(A)$, $A$ is isomorphic to $A_a \times A_{a'}$ and conversely if $A$ is isomorphic to $A_1 \times A_2$, then there exists an element $a \in B(A)$ such that $A_1$ is isomorphic to $A_a$ and $A_2$ is isomorphic to $A_{a'}$. Using the above decomposition theorem we prove that for any $a, b \in B(A)$ with $a \land b = F$, $A_a$ is isomorphic to $A_b$ if and only if there exists an isomorphism on $A$ which sends $a$ to $b$.

1. Preliminaries

First, we recall the definition of a $C$-algebra and some results, which will be used in the later text.
2  Decompositions of a C-algebra

By a C-algebra we mean an algebra of type (2,2,1) with operations \( \land, \lor, \) and \( ' \) satisfying the following properties:

(a) \( x'' = x; \)
(b) \( (x \land y)' = x' \lor y' ; \)
(c) \( (x \land y) \land z = x \land (y \land z); \)
(d) \( x \land (y \lor z) = (x \land y) \lor (x \land z); \)
(e) \( (x \lor y) \land z = (x \land z) \lor (x' \lor y \land z); \)
(f) \( x \lor (x \land y) = x; \)
(g) \( (x \land y) \lor (y \land x) = (y \land x) \lor (x \land y). \)

Clearly, every Boolean algebra is a C-algebra. The set \( \{T,F,U\} \) is a C-algebra with operations \( \land, \lor, \) and \( ' \) given by

\[
\begin{array}{c|ccc}
\land & T & F & U \\
\hline
T & T & F & U \\
F & F & F & F \\
U & U & U & U \\
\end{array}
\quad
\begin{array}{c|ccc}
\lor & T & F & U \\
\hline
T & T & T & T \\
F & T & F & U \\
U & U & U & U \\
\end{array}
\quad
\begin{array}{c|c}
X' & T & F \\
\hline
T & F & T \\
U & U & U \\
\end{array}
\]

We denote this three-element C-algebra by \( C \) and the two-element C-algebra (Boolean algebra) \( \{T,F\} \) by \( B \). It can be observed that the identities (a), (b) imply that the variety of all C-algebras satisfies the dual statements of (b) to (g). In general \( \land \) and \( \lor \) are not commutative in \( C \) and the ordinary right distributive law of \( \land \) over \( \lor \) fails in \( C \).

The following properties of a C-algebra can be verified directly [1, 3]:

(i) \( x \land x = x; \)
(ii) \( x \land y = x \land (x' \lor y) = (x' \lor y) \land x; \)
(iii) \( x \lor (x' \land x) = (x' \land x) \lor x = x; \)
(iv) \( (x \lor x') \land y = (x \land y) \lor (x' \land y); \)
(v) \( x \lor x' = x' \lor x; \)
(vi) \( x \lor y \lor x = x \lor y; \)
(vii) \( x \land x' \land y = x \land x'. \)

If a C-algebra \( A \) has an identity for \( \land \), then it is unique and we denote it by \( T \). In this case, we say that \( A \) is a C-algebra with \( T \). If we write \( F \) for \( T' \), then \( F \) is the identity for \( \lor \).

In a C-algebra, we have the following [1, 3]:

(viii) \( x \lor y = F \) if and only if \( x = y = F; \)
(ix) \( x \lor y = T \), then \( x \lor x' = T; \)
(x) \( x \lor T = x \lor x'; \)
(xi) \( T \lor x = T \) and \( F \land x = F; \)
(xii) \( a \in A, a' = a \) if and only if \( a \) is left zero of both \( \land \) and \( \lor \).

If there exists an element \( x \) in \( A \) such that \( x' = x \), then it is unique and we denote it by \( U \) (\( U \) is called the uncertain element of \( A \)). An element \( x \in A \) is called a central element of \( A \) if \( x \lor x' = T \). The set \( \{x \in A/ x \lor x' = T\} \) of all central elements of \( A \) is called the centre of \( A \) and is denoted by \( B(A) \). The set \( B(A) \) of all central elements of \( A \) is a Boolean algebra with respect to the operations \( \lor, \land, \) and \( ' \) (of \( A \)) restricted to \( B(A) \) [3].
For $x \in A$ define the relation $\theta_x$ on $A$ by $\theta_x = \{(p, q) \in A \times A / x \wedge p = x \wedge q\}$ then $\theta_x$ is a congruence relation on $A$ and $\theta_x \land \theta_x = \theta_x \lor \theta_x$ [1].

The relation $\leq$ defined on a $C$-algebra $A$ by $x \leq y$ if $y \land x = x$ is a partial ordering on $A$ in which, for every $x \in A$, the supremum of $\{x, x'\} = x \lor x'$, and the infimum of $\{x, x'\} = x \land x'$ [2]. If $A$ is a $C$-algebra with $T$, $x \in B(A)$ and $y \in A$ are such that $x \land y = y \land x$, then $x \lor y$ is the lub of $\{x, y\}$ and in this case $y \lor x$ need not be the lub of $x$ and $y$. For example, in the algebra $C, T \in B(C)$ and $T \land U = U \land T$ but $U \lor T = U$ is not the lub of $\{U, T\}$. If $x \leq y$, then $y \land x = x$ and hence $x \land y = x \land y \land x = x \land x = x$. Therefore $x \leq y$ if and only if $y \land x = x \land y$.

2. The $C$-algebra $A_x$

Recall that for every Boolean algebra $B$ and $a \in B$ the set $\{a\} = \{x \in B/x \leq a\}(\{a\} = \{x \in B/a \leq x\})$ is a Boolean algebra under the induced operations $\land$ and $\lor$ where complementation is defined by $x^* = a \land x'(x^* = a \lor x')$.

In this section, we prove that if $A$ is a $C$-algebra and $x \in A$, then $A_x = \{s \in A/s \leq x\}$ is a $C$-algebra with $T(= x)$ under the induced operations and that $A_x$ is isomorphic to a quotient algebra of $A$.

**Theorem 2.1.** Let $A$ be a $C$-algebra, $x \in A$, and $A_x = \{s \in A/s \leq x\}$. Then $(A_x, \land, \lor, *)$ is a $C$-algebra with $T$ where $\land$ and $\lor$ are the operations in $A$ restricted to $A_x$, $s^*$ is defined by $x \land s'$, and “$x$” is the identity for $\land$.

**Proof.** Clearly $A_x$ is closed under $\land$ and $\lor$. If $s \in A_x$, then $x \land s^* = x \land (x \land s') = (x \land x) \land s' = x \land s = s^*$. So that $s^* \in A_x$ and $s^* = (s^*)^* = (s^*)^* = x \land (x \land s') = x \land (x' \lor s) = x \land s = s$ (since $s \leq x$).

Now, for $s, t \in A_x$, $(s \land t)^* = x \land (s \land t)' = x \land (s' \lor t') = (x \land s') \lor (x \land t') = s^* \lor t^*$.

Finally, for $s, t, u \in A_x$,

\[
(s \lor t) \land u = x \land ((s \lor t) \land u) = x \land ((s \land u) \lor (s' \land t \land u)) \\
= ((x \land s) \land (x \land u)) \lor (x \land s' \land t \land u)
\]

\[
= (s \land u) \lor (s^* \land t \land u).
\]

The remaining identities hold in $A_x$ since they hold in $A$.

Hence $(A_x, \land, \lor, *)$ is a $C$-algebra with “$x$” as the identity for $\land$. □

Observe that $A_x$ is itself a $C$-algebra but it is not a subalgebra of $A$ because the unary operation $*$ is not the restriction of $'$ to $A_x$. Now, we give some properties of $A_x$.

**Theorem 2.2.** Let $A$ be a $C$-algebra. Then the following hold:

(i) $A_x = \{x \land s/s \in A\}$;

(ii) $A_x = A_y$ if and only if $x = y$;

(iii) $A_x \cap A_y \subseteq A_{x \land y}$;

(iv) $A_x \cap A_y = A_{x \land y}$;

(v) $(A_x)_{x \land y} = A_{x \land y}$. 
4 Decompositions of a C-algebra

Proof. (i), (ii), and (iii) can be verified routinely. We prove (iv) as follows. Let \( s \in A_{x \land x'} \), then \( (x \land x') \land s = s \) and hence \( x \land s = x \land (x \land x' \land s) = x \land x' \land s = s \). Also we have \( x' \land s = x' \land (x \land x' \land s) = s \). Since \( x \land x' = x' \land x \). Now we prove (v),

\[
(A_x)_{x \land y} = \{ x \land y \land t/t \in A_x \} \quad \text{by (i)}
\]
\[
= \{ x \land y \land x \land s/s \in A \}
\]
\[
= \{ x \land y \land s/s \in A \} = A_{x \land y}.
\]

\[\square\]

Let \( A_1, A_2 \) be two C-algebras with \( T_1 \) and \( T_2 \). Then a mapping \( f : A_1 \to A_2 \) that preserves \( \land, \lor, ' \) and carries \( T_1 \) to \( T_2 \) is called a \( T \)-preserving C-algebra homomorphism. In future, we deal with C-algebras with \( T \) only and hence by a C-algebra homomorphism we mean a \( T \)-preserving C-algebra homomorphism. The following lemma can be verified routinely.

**Lemma 2.3.** Let \( f : A_1 \to A_2 \) be a C-algebra homomorphism where \( A_1, A_2 \) are C-algebras with \( T_1 \) and \( T_2 \). Then

(i) if \( A_1 \) has the uncertain element \( U \), then \( f(U) \) is the uncertain element of \( A_2 \);

(ii) if \( a \in B(A_1) \), then \( f(a) \in B(A_2) \). The converse holds if \( f \) is one-one.

Now we prove the following.

**Theorem 2.4.** Let \( A \) be a C-algebra with \( T \) and \( x \in A \), then the mapping \( \alpha_x : A \to A_x \)

defined by \( \alpha_x(s) = x \land s \) for all \( s \in A \) is a homomorphism of \( A \) onto \( A_x \) with kernel \( \theta_x \) and hence \( A/\theta_x \cong A_x \).

Proof. For \( s \in A, x \land s \leq x \) and hence \( x \land s \in A_x \). Let \( s, t \in A \), then

\[
\alpha_x(s \land t) = x \land s \land t = x \land s \land x \land t = \alpha_x(s) \land \alpha_x(t),
\]
\[
\alpha_x(s') = x \land s' = x \land (x' \lor s') \quad \text{(by (ii) in the preliminaries)}
\]
\[
= x \land (x \land s') = (x \land s)' = (\alpha_x(s))'.
\]

Clearly, \( \alpha_x(s \lor t) = \alpha_x(s) \lor \alpha_x(t) \) and \( \alpha_x(T) = a \). Hence \( \alpha_x \) is a C-algebra homomorphism. Now, for \( s \in A_x \), we have \( \alpha_x(s) = s \). Therefore \( \alpha_x \) is onto homomorphism. Hence by the fundamental theorem of homomorphism \( A/\ker \alpha_x \cong A_x \) and \( \ker \alpha_x = \{(s,t) \in A \times A/\alpha_x(s) = \alpha_x(t) \} = \{(s,t) \in A \times A/x \land s = x \land t \} = \theta_x \). Thus \( A/\theta_x \cong A_x \).

\[\square\]

3. Decompositions of \( A \)

If \( B \) is a Boolean algebra and \( a \in B \), then we know that \( B \) is isomorphic to \( (a) \times (a) \). In this section we prove similar decompositions for a C-algebra. If \( A \) is a C-algebra with \( T \) and \( a \in B(A) \), then we prove that \( A \) is isomorphic to \( A_a \times A_{a'} \) and conversely. We also prove that if \( a, b \in B(A) \) and \( a \land b = F \), then \( A_a \) is isomorphic to \( A_b \) if and only if there is an automorphism on \( A \) that carries \( a \) to \( b \). First we prove the following.
Lemma 3.1. Let $A$ be a $C$-algebra with $T$, $a \in B(A)$ and $x, y \in A$. Then

$$a \lor x = a \lor y, \quad a' \lor x = a' \lor y \iff x = y. \quad (3.1)$$

Proof. Let $a \in B(A)$ and $x, y \in A$. Assume that $a \lor x = a \lor y$ and $a' \lor x = a' \lor y$. Then

$$x = F \lor x = (a \land a') \lor x = (a \lor x) \land (a' \lor x) = (a \lor y) \land (a' \lor y) = F \lor y = y. \quad (3.2)$$

The converse is trivial \hfill \Box

Note that Lemma 3.1 fails if $a / \in B(A)$. For example, in the $C$-algebra $C$, we have $U / \in B(C), U \lor T = U \lor F = U$, and $U' \lor T = U' \lor F = U$, but $T \neq F$.

Now we prove the following decomposition theorem.

Theorem 3.2. If $A$ is a $C$-algebra with $T$ and $a \in B(A)$, then $A \cong A_a \times A_{a'}$.

Proof. Define $\alpha : A \to A_a \times A_{a'}$ by

$$\alpha(x) = (\alpha_a(x), \alpha_{a'}(x)) \quad \forall x \in A. \quad (3.3)$$

Then, by Theorem 2.4, $\alpha$ is well defined and $\alpha$ is a homomorphism.

Now, $\alpha(x) = \alpha(y) \Rightarrow a \land x = a \land y$ and $a' \land x = a' \land y$. Hence $x = y$ (by the dual of Lemma 3.1). Finally, we prove $\alpha$ is onto. Let $(x, y) \in A_a \times A_{a'}$. Then $x \leq a$ and $y \leq a'$. So that $a \land x = x$ and $a' \land y = y$.

Thus, $a' \land x = a' \land a \land x = F$ and $a \land y = a \land a' \land y = F$.

Now,

$$x \lor y \in A, \quad \alpha(x \lor y) = (\alpha_a(x \lor y), \alpha_{a'}(x \lor y))$$

$$= (a \land (x \lor y), a' \land (x \lor y))$$

$$= ((a \land x) \lor (a \land y), (a' \land x) \lor (a' \land y))$$

$$= (x \lor F, F \lor y) = (x, y). \quad (3.4)$$

Hence $\alpha$ is an isomorphism. \hfill \Box

Now we prove the converse of the above theorem in the following sense.

Theorem 3.3. Let $A, A_1, A_2$ be $C$-algebras with $T$ such that $A \cong A_1 \times A_2$. Then there exists an element $a \in B(A)$ such that

$$A_1 \cong A_a, \quad A_2 \cong A_{a'}. \quad (3.5)$$
6 Decompositions of a C-algebra

Proof. Let \( \phi : A \to A_1 \times A_2 \) be an isomorphism and \( a = \phi^{-1}(T_1, F_2) \) (when \( T_1, T_2 \) denote the \( \wedge \)-identities of \( A_1, A_2 \), resp.)

Now \( (T_1,F_2) \in B(A_1 \times B(A_2) = B(A_1 \times A_2) \) and hence \( a \in B(A) \).

Define \( f : A_1 \to A_a \) by \( f(x_1) = \phi^{-1}(x_1, F_2) \) for all \( x_1 \in A_1 \).

Now

\[
a \wedge \phi^{-1}(x_1, F_2) = \phi^{-1}(T_1, F_2) \wedge \phi^{-1}(x_1, F_2)
\]

\[
= \phi^{-1}(x_1, F_2) \quad \text{(since } \phi^{-1} \text{ is a homomorphism).}
\]

Therefore \( \phi^{-1}(x_1, F_2) \in A_a \). Thus \( f \) is well defined.

It can be routinely verified that \( f \) preserves \( \wedge, \vee \) and that \( f \) is one-one.

Now we prove that \( f \) preserves the unary operation \( ' \).

Let \( x_1 \in A_1 \), then

\[
f(x_1') = \phi^{-1}(x_1', F_2) = \phi^{-1}(T_1 \wedge x_1', F_2 \wedge T_2)
\]

\[
= \phi^{-1}(T_1, F_2) \wedge \phi^{-1}(x_1', T_2) \quad \text{(since } \phi^{-1} \text{ is homomorphism)}
\]

\[
= a \wedge (\phi^{-1}(x_1, F_2))' = a \wedge f(x_1)' = (f(x_1))'.
\]

Finally, we prove \( f \) is onto.

Let \( x \in A_a \). Then \( \phi(x) = (x_1, x_2) \) for some \( x_1 \in A_1, x_2 \in A_2 \).

Now

\[
(x_1, x_2) = \phi(x) = \phi(a \wedge x) = \phi(a) \wedge \phi(x)
\]

\[
= (T_1, F_2) \wedge (x_1, x_2) = (x_1, F_2).
\]

Thus \( x_2 = F_2 \) and \( f(x_1) = \phi^{-1}(x_1, F_2) = \phi^{-1}(x_1, x_2) = x \).

Hence \( f \) is onto. Thus \( A_1 \cong A_a \). Similarly \( A_2 \cong A_{a'} \).

Finally, for \( a, b \in B(A) \) with \( a \wedge b = F \), we derive a necessary and sufficient condition for \( A_a \) to be isomorphic to \( A_b \). First we prove the following lemmas.

Lemma 3.4. If \( A \) is a C-algebra with \( T \), \( a \in B(A) \), \( x \in A_a \), and \( y \in A_{a'} \), then \( x \vee y = y \vee x \).

Proof. Let \( x \in A_a, y \in A_{a'} \). Then \( x \leq a \) and \( y \leq a' \). Hence \( a \vee y = F = a' \wedge x \). Now

\[
a \wedge (x \vee y) = (a \wedge x) \vee (a \wedge y) = x \vee F = x,
\]

\[
a \wedge (y \vee x) = (a \wedge y) \vee (a \wedge x) = F \vee x = x.
\]
Therefore, $a \land (x \lor y) = a \land (y \lor x)$. Similarly $a' \land (x \lor y) = a' \land (y \lor x)$.

By the dual of Lemma 3.1,

$$x \lor y = y \lor x. \tag{3.10}$$

\[ \square \]

**Lemma 3.5.** Let $A$ be a $C$-algebra with $T$. Then, for $a,b \in B(A)$, $a \land b \in B(A_a)$.

**Proof.** Clearly $a \land b \leq a$. Now

$$(a \land b) \lor (a \land b)^* = (a \land b) \lor (a \land (a \land b)')$$

$$= (a \land b) \lor [a \land (a' \lor b')] = (a \land b) \lor (a \land b') \tag{3.11}$$

$$= a \land (b \lor b') = a \land T = a.$$ 

Hence, $a \land b \in B(A_a)$. \[ \square \]

Now, we prove the theorem.

**Theorem 3.6.** Let $A$ be a $C$-algebra with $T$ and $a,b \in B(A)$ such that $a \land b = F$. Then $A_a$ is isomorphic to $A_b$ if and only if there exists an isomorphism $\alpha : A \to A$ such that $\alpha(a) = b$.

**Proof.** Let $a,b \in B(A)$ with $a \land b = F$. Let $\phi : A_a \to A_b$ be an isomorphism.

Now $a' \land b = (a' \land b) \lor F = (a' \land b) \lor (a \land b) = (a' \lor a) \land b = b$ because $B(A)$ is a Boolean algebra. So that $b \in A_{a'}$ and $b^* = a' \land b'$. Similarly, $b' \land a = a$. Now by Theorems 2.2, 3.2, and Lemma 3.5, we have

(i) $A \cong A_a \times A_{a'} \cong A_a \times A_{a'} \land b \times A_{(a' \land b)^*} = A_a \times A_b \times A_{a' \land b'}$

under the isomorphism $x \mapsto (a \land x, b \land x, (a' \land b') \land x)$;

(ii) $A \cong A_b \times A_{b'} \cong A_b \times A_{b'} \land a \times A_{(b' \land a)^*} \cong A_b \times A_a \times A_{a' \land b'}$

under the isomorphism $x \mapsto (b \land x, a \land x, (a' \land b') \land x)$;

(iii) $A_a \times A_b \times A_{a' \land b'} \cong A_b \times A_a \times A_{a' \land b'}$

under the isomorphism $(x,y,z) \mapsto (\phi(x),\phi^{-1}(y),z)$.

Now define $\alpha : A \to A$ by $\alpha = y^{-1} \circ \delta \circ \beta$. Then $\alpha$ is an isomorphism of $A$ onto $A$ and

$$\alpha(a) = (y^{-1} \circ \delta \circ \beta)(a) = y^{-1}(\delta(a,F,F)) \quad \text{(since } b \land a = F = a \land a')$$

$$= y^{-1}(b,F,F) \quad \text{(since } \phi(a) = b, \phi(F) = F) \tag{3.12}$$

$$= b \quad \text{(since } y(b) = (b,F,F)).$$

Hence $\alpha$ is an isomorphism of $A$ such that $\alpha(a) = b$.

Conversely, suppose that $\alpha : A \to A$ is an isomorphism such that $\alpha(a) = b$.

Let $\lambda$ be the restriction of $\alpha$ to $A_a$. Now we prove that $\lambda$ is an isomorphism of $A_a$ onto $A_b$. For $x \in A_a$,

$$b \land \lambda(x) = b \land \alpha(x) = \alpha(a) \land \alpha(x) = \alpha(a \land x) = \alpha(x) = \lambda(x). \tag{3.13}$$
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So that \( \lambda(x) \in A_b \). Hence \( \lambda \) is well defined. Clearly \( \lambda \) is a homomorphism and one-one. Let \( x \in A_b \). Since \( \alpha \) is onto, there exists \( y \in A \) such that \( \alpha(y) = x \). Now \( a \land y \in A_a \) and \( \lambda(a \land y) = \alpha(a \land y) = \alpha(a) \land \alpha(y) = b \land x = x \) (since \( x \leq b \)).

Hence \( \lambda \) is an isomorphism of \( A_a \) onto \( A_b \).

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Space dynamics is a very general title that can accommodate a long list of activities. This kind of research started with the study of the motion of the stars and the planets back to the origin of astronomy, and nowadays it has a large list of topics. It is possible to make a division in two main categories: astronomy and astrodynamics. By astronomy, we can relate topics that deal with the motion of the planets, natural satellites, comets, and so forth. Many important topics of research nowadays are related to those subjects. By astrodynamics, we mean topics related to spaceflight dynamics.

It means topics where a satellite, a rocket, or any kind of man-made object is travelling in space governed by the gravitational forces of celestial bodies and/or forces generated by propulsion systems that are available in those objects. Many topics are related to orbit determination, propagation, and orbital maneuvers related to those spacecrafts. Several other topics that are related to this subject are numerical methods, nonlinear dynamics, chaos, and control.

The main objective of this Special Issue is to publish topics that are under study in one of those lines. The idea is to get the most recent researches and published them in a very short time, so we can give a step in order to help scientists and engineers that work in this field to be aware of actual research. All the published papers have to be peer reviewed, but in a fast and accurate way so that the topics are not outdated by the large speed that the information flows nowadays.

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