ON THE EXISTENCE OF A PERIODIC SOLUTION OF A NONLINEAR ORDINARY DIFFERENTIAL EQUATION

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(Received March 8, 1996 and in revised form December 10, 1996)

ABSTRACT. Consider a planar forced system of the following form

\[ \begin{align*}
\frac{dx}{dt} &= \mu(x,y) + h(t) \\
\frac{dy}{dt} &= -\nu(x,y) + g(t),
\end{align*} \]

where \( h(t) \) and \( g(t) \) are \( 2\pi \)-periodic continuous functions, \( t \in (-\infty, \infty) \) and \( \mu(x,y) \) and \( \nu(x,y) \) are continuous and satisfy local Lipschitz conditions. In this paper, by using the Poincaré’s operator we show that if we assume the conditions, \( (C_1), (C_2) \) and \( (C_3) \) (see Section 2), then there is at least one \( 2\pi \)-periodic solution. In conclusion, we provide an explicit example which is not in any class of known results.

KEY WORDS AND PHRASES: Poincaré mapping, Poincaré index, Nonlinear jumping differential equation.


1. INTRODUCTION

We consider the following planar forced system of ordinary differential equations:

\[ \begin{align*}
\frac{dx}{dt} &= \mu(x,y) + h(t) \\
\frac{dy}{dt} &= -\nu(x,y) + g(t),
\end{align*} \] (1.1)

where \( h(t) \) and \( g(t) \) are \( 2\pi \)-periodic continuous functions, \( t \in (-\infty, \infty) \) and \( \mu(x,y) \) and \( \nu(x,y) \) are continuous and satisfy local Lipschitz conditions. Our work is inspired by previous work on the existence of periodic solutions of nonlinear ordinary differential equations by Aguinaldo and Schmitt [1], Cesari [4], Ding
In this paper, we show that if we assume the conditions, (C1), (C2) and (C3) (see Section 2), then there is at least one 2\pi-periodic solution. In conclusion, we provide an explicit example which is not in any class of known results.

The authors wish to take this opportunity to thank the referee for pointing out the wonderful paper Capietto et al. [2], which has results related to Theorem 2.1 as well as Lemma 2.1, and which both of us had overlooked.

2. MAIN RESULTS

Consider the system (1.1). Let \( x = \rho \cos \theta \) and \( y = \rho \sin \theta \). Expressing (1.1) in polar coordinates form, we have

\[
\begin{align*}
\frac{d\rho}{dt} &= \lambda(\rho, \theta) + \tilde{h}(\theta, t) \\
\frac{d\theta}{dt} &= -\omega(\rho, \theta) - \frac{\tilde{g}(\theta, t)}{\rho}
\end{align*}
\]

where

\[
\lambda(\rho, \theta) = \mu(\rho \cos \theta, \rho \sin \theta) \cos \theta - \nu(\rho \cos \theta, \rho \sin \theta) \sin \theta.
\]

\[
\omega(\rho, \theta) = \frac{1}{\rho}(\mu(\rho \cos \theta, \rho \sin \theta) \sin \theta + \nu(\rho \cos \theta, \rho \sin \theta) \cos \theta),
\]

\[
\tilde{h}(\theta, t) = h(t) \cos \theta + g(t) \sin \theta, \quad \text{and}
\]

\[
\tilde{g}(\theta, t) = h(t) \sin \theta - g(t) \cos \theta.
\]

We now shall establish the following theorem.

**THEOREM 2.1.** Suppose the system of differential equations (1.1) or (2.1) satisfies the conditions:

(C1) \[ |\lambda(\rho, \theta)| < k(\rho), \] where \( k(\rho) \) is a continuous function and satisfies a local Lipschitz condition.

(C2) (a) \[
\frac{2\pi + \delta}{n+1} < \lim_{\rho \to \infty} \int_0^{2\pi} \frac{d\theta}{\omega(\rho, \theta)} \leq \lim_{\rho \to \infty} \int_0^{2\pi} \frac{d\theta}{\omega(\rho, \theta)} < \frac{2\pi - \delta}{n} \]

for some \( n \in \mathbb{Z}^+ \) and some \( \delta, \quad 0 < \delta < \min \{2\pi/(2n+1), 1\} \) or

(b) \[
2\pi + \delta < \lim_{\rho \to \infty} \int_0^{2\pi} \frac{d\theta}{\omega(\rho, \theta)} \leq \lim_{\rho \to \infty} \int_0^{2\pi} \frac{d\theta}{\omega(\rho, \theta)} < \frac{2\pi}{\delta} \]

for some \( \delta, \quad 0 < \delta < (\sqrt{n+1}-1) \).

(C3) \[
\lim_{\rho \to \infty} \min_{0 \leq \theta < 2\pi} \omega(\rho, \theta) \rho^\alpha > 0 \]

for some \( \alpha \in (0,1) \).

Then the system has at least one 2\pi-periodic solution. Take note that the assumptions of this theorem are comparable with the assumptions of Theorem 3 in [2, p.367].

We first establish two lemmas.

**LEMMA 2.1.** If the system (1.1) or (2.1) satisfies (C1), then for each positive number \( M_1 \), there exists a positive number \( M \), with \( M \geq M_1 \) so that for each solution \((\rho(t), \theta(t))\) of (2.1), with an initial condition \( \rho(t_0) > M \) for \( t_0 \in [0, 2\pi] \), we have \( \rho(t) > M_1 \) for \( 0 \leq t \leq 2\pi \).

This lemma is very close to the "elastic property" in [2, p.351].
PROOF. From (2.1) and (C1), we have
\[-c_1 - k(\rho) < \frac{d\rho}{dt} < k(\rho) + c_1\]
where
\[c_1 = \max_{0 \leq s < 2\pi} |\vec{h}(\theta, t)| + \varepsilon_0, \text{ where } \varepsilon_0 > 0.\]

Let \(\rho^+(t)\) and \(\rho^-(t)\) be solutions of the equations
\[\frac{d\rho}{dt} = k(\rho) + c_1 \quad (+)\]
and
\[\frac{d\rho}{dt} = -k(\rho) - c_1 \quad (-)\]
with the initial conditions \(\rho^+(t_0) = M_1\) for (+) and \(\rho^-(t_0) = \rho^+(2\pi)\) for (-).

Observe that if \(\rho^+(t)\) is a solution of (+) then \(\rho^+(2\pi + t_0 - t)\) will be a solution of (-). Thus \(\rho^-(t) = \rho^+(2\pi + t_0 - t)\) with \(\rho^-(t_0) = \rho^+(2\pi)\). Let \(M = \rho^+(2\pi)\). Because \(-k(\rho) - c_1 < 0\), from (-), we have \(\rho^-(2\pi) \leq \rho^-(t)\) for all \(0 \leq t \leq 2\pi\). Let \(\rho(t)\) be a solution of \(\frac{d\rho}{dt} = \Lambda(\rho, \theta) + \vec{h}(\theta, t)\) with an initial condition \(\rho(t_0) > M\). Then, according to the comparison theorem (e.g., see [10, p.26]) of ordinary differential equations, we have \(\rho^-(t) < \rho(t)\), as \(0 \leq t \leq 2\pi\). Together, we have \(M = \rho^-(2\pi) \leq \rho^-(t) < \rho(t)\) for all \(t, 0 \leq t \leq 2\pi\). Observe that \(M = \rho^-(t_0) = \rho(2\pi) = M_1 > 0\), from the above inequality. The lemma is now proved.

**Lemma 2.2.** If the system (2.1) satisfies (C1), (C2), (a) or (b), and (C3) then there exists a constant \(M > 0\) such that for every solution \((\rho(t), \theta(t))\) of (2.1) with an initial condition \(\rho(t_0) > M\), for \(t_0 \in [0, 2\pi]\), the inequality:

\[
2n\pi < \theta(t) - \theta(2\pi) < 2(n+1)\pi, \quad \text{when } n \in \mathbb{Z}^+, \quad (2.2a)
\]

holds if (C2)(a) is assumed, and the inequality

\[
0 < \theta(t) - \theta(2\pi) < 2\pi \quad (2.2b)
\]

holds if (C2)(b) is assumed.

**PROOF.** By (C3) and (C2), (a) or (b), there is a positive constant \(M_2\) such that for all \(\rho > M_2\), we have \(\omega(\rho, \theta) > 0\),

\[
\omega(\rho, \theta) \rho^\alpha > \frac{c}{2}, \quad (2.3)
\]

for any \(c\), \(\lim_{\rho \to \infty} \min_{0 \leq \theta < 2\pi} \omega(\rho, \theta) \rho^\alpha > c > 0\), and

\[
\frac{2\pi + (\delta/2)}{n+1} < \int_0^{2\pi} \frac{d\theta}{\omega(\rho, \theta)} < \frac{2\pi - (\delta/2)}{n} \quad (2.4a)
\]

for some \(n \in \mathbb{Z}^+\) if (C2)(a) is assumed, or

\[
2\pi + \frac{\delta}{2} < \int_0^{2\pi} \frac{d\theta}{\omega(\rho, \theta)} < \frac{1}{\delta} \quad (2.4b)
\]

if (C2)(b) is assumed. Let
\( c_2 = \max_{0 \leq \theta \leq 2\pi} |g(\theta, t)| \)

where

\[
M_3 = \max \left\{ M_2, \left( \frac{\beta \rho_2 C_5}{C_6} \right)^{1-\alpha} \right\}.
\]  

By Lemma 2.1, there exists \( M > 0 \) such that for every solution \((p(t, \theta(t))\) of (2.1) with \( \rho(t_0) > M \) we have \( \rho(t) > M_3 \) for \( 0 \leq t \leq 2\pi \). Divide the second equation of (2.1) by \(-\omega(p, \theta)\) and integrate between \( \theta(0) \) and \( \theta(2\pi)\):

\[
- \int_{\theta(0)}^{\theta(2\pi)} \frac{d\theta}{\omega(p, \theta)} = \int_{0}^{2\pi} \frac{g(\theta, t)}{\omega(p, \theta)\rho} dt + \int_{0}^{2\pi} \frac{\sigma}{\omega(p, \theta)\rho} dt.
\]

Assume (2.2a) (or (2.2b)) is false. Then, say \( \theta(0) - \theta(2\pi) \geq 2(n+1)\pi \), or \( \theta(0) - 2(n+1)\pi = \theta(2\pi) \), where \( n \in \mathbb{Z}^+ \), or \( n = 0 \). By using (2.3), (2.4a) or (2.4b), (2.5), (2.6), we have:

\[
\frac{2\pi + \frac{\pi}{2}}{2(n+1)} \int_{\theta(0)-2\pi}^{\theta(0)} \frac{d\theta}{\omega(p, \theta)} = \int_{\theta(0)-2(n+1)\pi}^{\theta(0)} \frac{d\theta}{\omega(p, \theta)} < \int_{\theta(0)}^{\theta(2\pi)} \frac{d\theta}{\omega(p, \theta)}.
\]

However,

\[
\int_{\theta(0)}^{\theta(2\pi)} \frac{d\theta}{\omega(p, \theta)} = \int_{0}^{2\pi} \frac{g(\theta, t)}{\omega(p, \theta)\rho} dt < 2\pi + \frac{\pi}{2}.
\]

This is a contradiction. We must have \( \theta(0) - \theta(2\pi) < 2(n+1)\pi \), where \( n \in \mathbb{Z}^+ \) (or \( \theta(0) - \theta(2\pi) < 2\pi \)). Similarly we have \( \omega(0) - \omega(2\pi) > 2n\pi \), where \( n \in \mathbb{Z}^+ \) (or \( \omega(0) - \omega(2\pi) > 0 \)). The lemma is now proved.

As a direct consequence of Lemma 2.2, we have the following Corollary.

**Corollary 2.1.** If, in addition to the conditions of Lemma 2.2, we assume \( g(t) = h(t) = 0 \), then all \( 2\pi \)-periodic solutions of (2.1) are uniformly bounded, that is, there is a positive number \( M \) such that \( \rho(t) \leq M \) for every \( 2\pi \)-periodic solution \((p(t, \theta(t))\) of (2.1).

**Proof of Theorem 2.1.** Let us denote \((x(t; x_0, y_0), y(t; x_0, y_0))\) the solution of (1.1) with the initial condition \((x(0; x_0, y_0), y(0; x_0, y_0)) = (x_0, y_0)\). For every point \((x_0, y_0)\), define the Poincaré mapping \( P(x_0, y_0) = (x(2\pi; x_0, y_0), y(2\pi; x_0, y_0)) \). Since \( u(x, y), v(x, y), h(t) \) and \( g(t) \) of (1.1) are continuous, the Poincaré mapping \( P(x_0, y_0) \) of (1.1) is continuous. Let \( u(x_0, y_0) = x(2\pi; x_0, y_0) - x_0 \) and \( v(x_0, y_0) = y(2\pi; x_0, y_0) - y_0 \). The pair \((u, v)\) is a continuous vector field in the plane.

Let \( \beta \) be the positive constant guaranteed by Lemma 2.2. Choose a positive constant \( \beta' \) such that \( \beta' > \beta \). According to the Lemma 2.2, for all \((x_0, y_0)\) such that \( x_0^2 + y_0^2 = (\beta')^2 \), we have \( 2\pi < \theta(0) - \theta(2\pi) < 2(n+1)\pi \) for some nonnegative integer \( n \). Thus \( u(x_0, y_0), v(x_0, y_0) = \alpha(x_0, y_0) \) for any real constant \( \alpha \) and for all \((x_0, y_0)\) such that \( x_0^2 + y_0^2 = (\beta')^2 \). As \((x_0, y_0)\) moves along the circle \( x_0^2 + y_0^2 = (\beta')^2 \) in counter clockwise direction, the vector \( u(x_0, y_0), v(x_0, y_0) \) rotates through an angle of \( 2\pi \). The sum of the Poincaré index of a singular point of the continuous vector field \((u(x_0, y_0), v(x_0, y_0))\) in the compact domain = \( \{(x_0, y_0)|x_0^2 + \)}
y^2 \leq (\mathcal{M}^{'})^2 is equal to 1. There must exist at least one singular point of the vector field in this domain. We denote it by \((x, y), P(x, y) = (\bar{x}, \bar{y})\). It is clear that \(x^2 + y^2 \leq (\mathcal{M}^{'})^2\). Hence \((x(2\pi; x, y), y(2\pi; x, y)) = (\bar{x}, \bar{y})\). This is a 2\pi-periodic solution.

The following two theorems (2.2 and 2.3) are almost the same as two well-known theorems. However, these particular two theorems require slightly different assumptions than the two well-known theorems. Furthermore, these two theorems can be proved using Theorem 2.1.

**Theorem 2.2.** Consider the following nonlinear ordinary differential equation:

\[
\frac{d^2x}{dt^2} + (\omega_+)^2x^+ - (\omega_-)^2x^- = H(t),
\]

where \(\omega_+\) and \(\omega_-\) are positive constants, \(x^+ = \max\{x, 0\}, x^- = \max\{-x, 0\}\) and \(H(t)\) is a continuous 2\pi-periodic function. If

\[
\frac{2}{n+1} < \frac{1}{\omega_+} + \frac{1}{\omega_-} < \frac{2}{n}
\]

for some \(n \in \mathbb{Z}^+\) \[(2.8a)\]

or

\[
2 < \frac{1}{\omega_+} + \frac{1}{\omega_-} < \infty,
\]

\[(2.8b)\]

then the equation (2.7) has a 2\pi-periodic solution.

**Proof.** Let us define

\[
u(x) = \begin{cases} (\omega_+)^2 & \text{if } x \geq 0 \\ (\omega_-)^2 & \text{if } x < 0. \end{cases}
\]

Then

\[
u(x) \cdot x = (\omega_+)^2 x^+ - (\omega_-)^2 x^- = \begin{cases} (\omega_+)^2 x & \text{if } x \geq 0 \\ (\omega_-)^2 x & \text{if } x < 0. \end{cases}
\]

Clearly, \(\nu(x) \cdot x\) is a continuous function of \(x\).

Let \(\frac{dx}{dt} = y\). Then the equation (2.7) is equivalent to:

\[
\begin{align*}
\frac{dx}{dt} &= y \\
\frac{dy}{dt} &= -\nu(x) \cdot x + H(t)
\end{align*}
\]

which is of the same type as (1.1). It satisfies a local Lipschitz condition. The Lipschitz constant is equal to \(\max(\omega_+^2, \omega_-^2, 1)\). Expressing (2.9) in the polar coordinates, we have

\[
\begin{align*}
\frac{d\rho}{dt} &= \lambda(\rho, \theta) + H(t) \sin \theta \\
\frac{d\theta}{dt} &= -\omega(\rho, \theta) - \frac{1}{\rho} (-H(t) \cos \theta),
\end{align*}
\]

when \(\lambda(\rho, \theta) = \rho \sin \theta \cos \theta - \nu(x) \cdot \rho \sin \theta \sin \theta = \rho \sin \theta \cos \theta (1 - \nu(x))\)

\[
\begin{align*}
\rho \sin \theta \cos \theta (1 - \omega_+^2) & \quad \text{if } -\frac{\pi}{2} \leq \theta < \frac{\pi}{2} \\
\rho \sin \theta \cos \theta (1 - \omega_-^2) & \quad \text{if } \frac{\pi}{2} \leq \theta < \frac{3\pi}{2}
\end{align*}
\]
and

\[ \omega(\rho, \theta) = \sin^2 \theta + \frac{1}{\rho} \mu(x) \cdot \cos \theta \]

\[ = \sin^2 \theta + \mu(x) \cos^2 \theta = \begin{cases} 
\sin^2 \theta + \omega^2 \cos^2 \theta, & \text{if } -\frac{\pi}{2} \leq \theta < \frac{\pi}{2} \\
\sin^2 \theta + \omega^2 \cos^2 \theta, & \text{if } \frac{\pi}{2} \leq \theta < \frac{3\pi}{2}.
\end{cases} \]

Now,

\[ \int_0^{2\pi} \frac{d\theta}{\omega(\rho, \theta)} = \pi \left[ \frac{1}{\omega^+} + \frac{1}{\omega^-} \right]. \]

It is easy to verify that the equation (2.7) satisfies the conditions (C1), (C2) and (C3).

**THEOREM 2.3.** Consider

\[ \frac{d^2x}{dt^2} + G(x) = H(t) \quad (2.11) \]

where \( G(x) \) is Lipschitz continuous with respect to \( x \), and \( H(t) \) is \( 2\pi \)-periodic and continuous with respect to \( t \). Suppose (2.11) satisfies the following condition:

\[ (n+\delta)^2 < \lim_{|x| \to \infty} \frac{G(x)}{x} \leq \lim_{|x| \to \infty} \frac{G(x)}{x} < (n+1-\delta)^2, \quad (2.12) \]

for some non-negative integer \( n \) and some \( \delta \),

\[ 0 < \delta < \min \left\{ \frac{2\pi}{2n+1}, \frac{1}{4\pi} \right\} = \min \left\{ \frac{2\pi}{2n+1}, \frac{1}{4\pi} \right\}. \]

Then the equation (2.11) has a \( 2\pi \)-periodic solution.

**PROOF.** Let \( \frac{dx}{dt} = y \), then the equation (2.11) is equivalent to the system

\[ \begin{cases} 
\frac{dx}{dt} = y \\
\frac{dy}{dt} = -G(x) + H(t). 
\end{cases} \quad (2.13) \]

Observe that the system (2.13) is a special case of the system (1.1), where \( \mu(x, y) = y, \nu(x, y) = G(x), h(t) = 0 \) and \( g(t) = H(t) \). Expressing (2.11) in the polar coordinates form (2.1), we have: \( \lambda(\rho, \theta) = \rho \cos \theta - G(\rho \cos \theta) \sin \theta, \omega(\rho, \theta) = \sin^2 \theta + \frac{1}{\rho} G(\rho \cos \theta) \cos \theta, \tilde{h}(\theta, t) = H(t) \sin \theta \) and \( \tilde{g}(\theta, t) = -H(t) \cos \theta \).

Now, one can easily verify that the equation (2.13) satisfies the conditions (C1), (C2), and (C3).

3. **EXAMPLE**

Consider the system

\[ \begin{cases} 
\frac{dx}{dt} = y \\
\frac{dy}{dt} = -f(x, y) + H(t) 
\end{cases} \quad (3.1) \]

where

\[ f(x, y) = \begin{cases} 
0 & \text{if } (x, y) = (0, 0) \\
\frac{k^2 x^3}{x^2 + y^2} & \text{otherwise,}
\end{cases} \]
k > \frac{1}{2} \text{ and } H(t) \text{ is a continuous } 2\pi\text{-periodic function. Then (3.1) has a } 2\pi\text{-periodic solution.}

\textbf{Proof.} Observe that the function } f(x,y) \text{ is continuous, and } \frac{\partial f}{\partial x} \text{ and } \frac{\partial f}{\partial y} \text{ exist, for all } x,y. \text{ We now show that (3.1) satisfies the conditions (C1), (C2), and (C3).}

Express (3.1), in polar coordinate form. Then

\begin{align*}
\frac{dp}{dt} & = \rho \sin \theta \cos \theta (1 - k^2 \cos^2 \theta) + H(t) \sin \theta \\
\frac{d\theta}{dt} & = (\sin^2 \theta + k^2 \cos^4 \theta) + H(t) \cos \theta
\end{align*}

with \( \lambda(p, \theta) = \rho \sin \theta \cos \theta (1 - k^2 \cos^2 \theta) \) and \( \omega(p, \theta) = \sin^2 \theta + k^2 \cos^4 \theta. \)

Using the Residue Theorem of complex analysis we find

\begin{align*}
\int_{-\pi/2}^{\pi/2} \frac{d\theta}{\omega(p, \theta)} & = 2 \int_{-\pi/2}^{\pi/2} \frac{d\tan \theta}{\tan^2 \theta + \frac{k^2}{1 + \tan^2 \theta}} \\
& = 2 \int_{-\infty}^{\infty} \frac{(1 + x^2) \, dx}{(x^2 + \sqrt{2k-1} \, x + k)(x^2 + 2k-1 \, x + k)}
\end{align*}

There are two complex roots:

\( x_1 = \frac{-\sqrt{2k-1} + i\sqrt{2k+1}}{2} \)

and

\( x_2 = \frac{\sqrt{2k-1} + i\sqrt{2k+1}}{2} \)

in the upper-half complex plane. Thus the above integral is equal to

\begin{align*}
4\pi \int \frac{1 + x_1^2}{(x_1 - x_1)(x_1 - \sqrt{2k-1} \, x_1 + k)} + \frac{1 + x_2^2}{(x_2 - x_2)(x_2 + \sqrt{2k-1} \cdot x_2 + k)}
\end{align*}

\begin{align*}
= 4\pi \frac{4(2k-1)(k+1)}{2i\sqrt{2k+1} \, (4k(2k-1))} = \frac{2\pi(k+1)}{k\sqrt{2k+1}}.
\end{align*}

Since \( k \) and \( k + 1 \) are relatively prime and \( k + 1 > \sqrt{2k+1} \) as \( k \geq \frac{1}{2} \), it follows that \( \frac{k\sqrt{2k+1}}{k+1} \) cannot be an integer, and thus there exists a non-negative integer \( n \) such that \( n < \frac{k\sqrt{2k+1}}{k+1} < n + 1. \) The condition (C2), (a) or (b), is satisfied.

It is evident that (C1) and (C3) are true. In fact:

\( |\lambda(p, \theta)| = \rho |\sin \theta \cos \theta (1 - k^2 \cos^2 \theta)| < \rho (1 + k^2) = k(p) \)

and

\( \lim_{\rho \to \infty} \min_{0\leq \theta < 2\pi} \omega(p, \theta)^\alpha = \lim_{\rho \to \infty} \left( 1 - \frac{1}{4k^2} \right)^\alpha = \infty. \)

Hence, by Theorem 2.1, there is a \( 2\pi \)-periodic solution.
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