A SOURCE OF RESULTS IN PROJECTIVE GEOMETRY

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Abstract

Let $\Omega$ and $\Omega'$ be subsets of a real Euclidean plane $\mathcal{E}$ and let $\varphi: \Omega \rightarrow \Omega'$ be a bijection. Then $\varphi$ induces on $\Omega'$ a copy of the restriction to $\Omega$ of the geometry of $\mathcal{E}$. By considering the closures $\overline{\varphi(S \cap \Omega)}$ for sets $S \subset \mathcal{E}$, this induced geometry can be extended. This procedure is explored in the case of a projective transformation, and leads to results in real and complex projective planes which are analogues of Euclidean ones.

1. Introduction

Let $\mathcal{E}$ be a real Euclidean plane (see, e.g. [1, pp 1–21]). Let $\Omega$, $\Omega'$ be open subsets of $\mathcal{E}$ and consider a bijection $\varphi: \Omega \rightarrow \Omega'$. For any set $S \subset \mathcal{E}$, clearly

$$S' = \varphi(S \cap \Omega) \subset \Omega'$$

and, in the manner of the models in [13, pp 52–5] and [10, chapter 10], $\varphi$ will induce on $\Omega'$ an isomorphic copy of the restriction to $\Omega$ of the geometry of $\mathcal{E}$. We shall also consider the closure

$$S' = \overline{\varphi(S \cap \Omega)}$$

which we denote by $\varphi'(S)$.

In this paper, we study one example of such a function $\varphi$ and use it for the purpose of exploration. What emerges can be presented as a general programme as follows: to identify as far as possible the induced concepts and induced sets $S'$, and to link their properties to the relationships on the boundary $\partial \Omega'$ of sets of the form $\varphi'(S_1)$ and $\varphi'(S_2)$.

Throughout we shall use a fixed frame of reference, $\mathcal{F} = (O, I, [O, J])$, consisting of an ordered pair of half-lines, where the lines $OI$ and $OJ$ are perpendicular. For any point $P \in \mathcal{E}$ we take Cartesian coordinates $(x, y)$, where $OI$ is the $x$-axis with