ON X-KÖTHE ECHELON SPACES AND APPLICATIONS

By FERNANDO BLASCO
U. D. Matématicas, ETSI Montes, Universidad Politécnica de Madrid

(Communicated by S. Dineen, M.R.I.A.)

[Received 18 June 1997. Read 29 January 1998. Published 30 December 1998.]

ABSTRACT

Properties such as Heinrich’s density condition or being Montel for X-Köthe echelon spaces are considered and applications to the study of reflexivity of spaces of n-homogeneous continuous polynomials on Fréchet spaces are given.

Introduction

In this article we study a class of Fréchet spaces, the X-Köthe echelon spaces, that generalises the class of Köthe echelon spaces $\ell_p(A)$. The spaces $\ell_p(A)$, $1 \leq p \leq \infty$, and many of their properties have been widely studied (see, for instance, [3], [4], [22], [30]) because they are a good source of examples and counterexamples in functional analysis. The spaces studied here are more general than Köthe echelon spaces and are also a good source of getting examples, as we shall see in §4. Bellenot defines X-Köthe echelon spaces and obtains some of their properties in [2]. This ‘new’ class of echelon spaces has already been used to give examples and counterexamples [27]. We describe properties of X-Köthe echelon spaces, point out similarities with ‘classical’ Köthe echelon spaces and give applications. The paper consists of four sections: in §1 we study basic properties of $\mathcal{X}_A(A)$ and describe the dual space. In §2 we characterise X-Köthe echelon spaces that have Heinrich’s density condition. Section 3 describes $\mathcal{X}_A(A)$ spaces that are Montel in terms of their Köthe matrix. Finally, in §4, examples and applications are given, which show the importance of these spaces in constructing new examples in functional analysis.

1. Basic properties

Let $X$ be a Banach space with a basis $\{e_n\}_{n=1}^\infty$ and let $a = (a_n)_{n\in\mathbb{N}}$ be a sequence of positive numbers. Suppose that $\| \cdot \|$ denotes the norm in $X$. We define

$$X_a = \left\{(x_n)_{n\in\mathbb{N}} \in \mathbb{C}^\mathbb{N} : \sum_{n=1}^\infty a_n x_n e_n \in X \right\}$$

AMS Subject Class. (1991): 46G20, 46A03