NAMBU DYNAMICS, n-LIE ALGEBRAS AND INTEGRABILITY

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Abstract. We present a generalized formulation of Poisson dynamics suitable to describe the n-bodies interactions. Examples are given of physical systems endowed with such a general structure.

1. Introduction

The Nambu dynamics is an example of n-Poisson structure which is a special n-Lie algebra. The latter was introduced for the first time by V. T. Filippov [4] in 1985 who gave first examples, developed first structural concepts, like simplicity, in this context and classified n-Lie algebras of dimensions n + 1 which is parallel to the Bianchi classification of three-dimensional Lie algebras.

Filippov defines an n-Lie algebra structure to be an n-ary multi-linear and anti-symmetric operation

\[ [v_1, \ldots, v_{n-1}] \]

which satisfies the n-ary Jacobi identity:

\[ [v_1, \ldots, v_{n-1}, [u_1, \ldots, u_n]] = \sum_{i=1}^{n} [u_1, \ldots, u_{i-1}, [v_1, \ldots, v_{n-1}, u_i], u_{i+1}, \ldots, u_n] \]

(1)

Such an operation, realized on the smooth function algebra of a manifold and additionally assumed to be an n-derivation, is an n-Poisson structure.

This general concept, however, was not introduced by Filippov. A first proposal goes back to M. L. Albeggiani (1936) [13] who introduced, in a different context,