NETS OF ASYMPTOTIC LINES IN A RIEMANNIAN HYPERSURFACE WITH NON-SYMMETRIC METRIC CONNECTION

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Abstract. If \( M \) is a Riemannian space we denote by \( g \) its Riemannian metric and by \( \nabla \) its Riemannian connection. Additionally we assume that \( M \) admits a non-symmetric metric connection \( \nabla^* \). Let \( C: x^i = x^i(s) \) be a curve in \( M_n(\nabla, g) \) parametrized by the arc length. This curve can also be considered to be in \( M_n(\nabla^*, g) \). Using the normal curvature of \( C \) in \( M_n(\nabla^*, g) \), one can obtain the asymptotic lines of \( M_n(\nabla^*, g) \).

A curve in a hypersurface is defined to be an asymptotic line if the normal curvature along the curve vanishes identically. Here we investigate the special nets which are formed by the tangent fields of asymptotic lines in \( M_n(\nabla^*, g) \) from that of the nets in \( M_n(\nabla, g) \) and by using the coefficients of \( \tilde{\nabla}^* \) and \( \tilde{\nabla} \).

1. Introduction

Let \( M \) be an \( n \)-dimensional Riemannian space having a symmetric connection \( \nabla \) and let us denote by \( g_{ij} \) and \( \{^i_{jk}\} \) the metric connection and the Christoffel symbols formed with the help of \( g_{ij} \). Such a Riemannian space will be denoted by \( M_n(\nabla, g) \).

Let \( v^i_r \) \((r = 1, 2, \ldots, n)\) be the contravariant components of the \( n \) independent vector fields \( \vec{v} \) in \( M_n \) which satisfy the condition \( g_{ij} v^i_r v^j_r = 1 \).

Following [1], we defined the covector fields \( \tilde{v} \) satisfying the equalities

\[
\begin{align*}
    v^i_r \tilde{v}_j &= \delta^i_j, \\
    v^i_r \tilde{v}_j^p &= \delta^p_r \\
    i, j, r, p &= 1, 2, \ldots, n.
\end{align*}
\]