NECESSARY CONDITIONS FOR A SUPERDIFFERENTIABLE SUPERCURVE TO BE A WEAK MINIMUM RELATIVE TO TWO SUB-SUPERMANIFOLDS

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Abstract. Let $L$ defines a regular problem in the calculus of variations on supermanifolds. The necessary conditions for a piecewise superdifferentiable supercurve $C$ in sense of Rogers to be a weak local minimum relative to two sub-supermanifolds are given.

Let $V$ be a supervector space [3], $V^*$ be the dual supervector space [5], $M$ be a supermanifold in the sense of Rogers [7] and $T(M)$ be the tangent superspace or superbundle [5] over $M$.

Let us consider only algebras over the real numbers. For each positive integer $L$, $B_L$ [7] will denote the Grassmannian algebra over the real numbers with generators $1^L, \beta_1^L, \ldots, \beta_L^L$ and relations

\[ 1^L \cdot \beta_i^L = \beta_i^L \cdot 1^L = \beta_i^L, \quad i = 1, \ldots, L, \]
\[ \beta_i^L \cdot \beta_j^L = -\beta_j^L \cdot \beta_i^L, \quad i, j = 1, \ldots, L. \]

$B_L$ is a graded algebra [8] and can be written as a direct sum [7]

\[ B_L = (B_L)_0 \oplus (B_L)_1 \]

where $(B_L)_0$ and $(B_L)_1$ are the even and the odd parts of $(B_L)$ respectively. We consider the $(m, n)$-dimensional supereuclidean space $B_L^{m,n} = (B_L)_0 \oplus (B_L)_1$ [7] with $L > n$. Let $M_{\|}^L$ denote (following Kostant [6]) the set of finite sequences of positive integers $\mu = (\mu_1, \ldots, \mu_k)$ with $1 \leq \mu_1 < \cdots < \mu_k \leq L$. $M_{\|}^L$ includes also the sequence with no elements, which is denoted by $\phi$. As it follows from [6] for each $\mu$ in $M_{\|}^L$

\[ \beta^{(L)}_{\mu} = \beta^{(L)}_{\mu_1} \cdots \beta^{(L)}_{\mu_k}, \quad k = 1, \ldots, L \]