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$\mathfrak{g}$-SYMPLECTIC ORBITS AND A SOLUTION OF THE BRST CONSISTENCY CONDITION

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Abstract. For any Lie algebra $\mathfrak{g}$ we introduce the notion of $\mathfrak{g}$-symplectic structures and show that every orbit of a principal G-bundle carries a natural $\mathfrak{g}$-symplectic form and an associated momentum map induced by the Maurer–Cartan form on G. We apply this to the BRST bicomplex and show that the associated momentum map is a solution of the Wess–Zumino consistency condition for the anomaly.

1. Introduction

We first introduce the notion of Lie algebra $\mathfrak{g}$-valued symplectic structures and we show that every orbit of a principal G-bundle carries a natural $\mathfrak{g}$-symplectic form, which is exact and induced from the Maurer–Cartan form on the Lie group G. The G-action has a natural momentum map which is an invariant for any fundamental vector field. In order to give a solution to the BRST (Wess–Zumino) consistency condition, we generalize these results to infinite dimensional group $\mathcal{G}$ of gauge transformations which acts on $\mathfrak{g}$-valued differential forms. On these orbit spaces we have the natural $\mathfrak{g}$-valued 1-form Θ, induced by the Maurer–Cartan form on the Lie group $\mathcal{G}$, and the corresponding momentum map. We summarize the classical BRST transformations described as coboundary operator of the Chevalley–Eilenberg complex of the infinite dimensional Lie algebra $\mathfrak{g}$ of infinitesimal gauge transformations, [10–12]. Next we describe the chiral anomaly as element of the first cohomology of the local BRST complex [11, 12] using an induced representation of $\mathfrak{g}$ on local forms. We consider the Wess–Zumino consistency condition as a problem in this BRST cohomology. To find a solution we combine the BRST bicomplex with the idea of $\mathfrak{g}$-valued symplectic geometry and momentum maps. We show that