FINITE GROUP ACTIONS IN SEIBERG–WITTEN THEORY

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Abstract. Let $X$ be a closed and oriented Riemannian four-manifold with $b_2^+(X) > 1$. We discuss the Seiberg–Witten invariants of $X$ and finite group actions on spin$^c$ structures of $X$. We introduce and comment some of our results on the subject.

1. Introduction

In the past twenty years, the symbiosis between mathematics and theoretical physics has always been a source of unexpected and profound results. Even if we do not make attempt to relate it chronologically, the story begun with the Donaldson’s gauge theory aiming a nonabelian generalization of the classical electromagnetic theory. As results of it the nonsmoothability of certain topological four-manifolds, exotic smooth structures on $\mathbb{R}^4$, and nondecomposability of some four-manifolds have been established. The computation of Donaldson invariants however is highly nontrivial. In 1994, the monopole theory in four-manifolds gave a rise to the Seiberg–Witten invariant which is much simpler than the Donaldson theory, also had almost the same effects on the Donaldson theory, and was used for a proof of the Thom conjecture. At almost the same time the Gromov–Witten invariant of symplectic manifolds was introduced. Using it we may compute the number of algebraic curves, representing a two-dimensional homology class in a symplectic manifold. In 1995 Taubes [26] proved that for symplectic four-manifolds the Seiberg–Witten invariant and the Gromov–Witten invariant are the same. In 1982 Freedman [15] classified the simply connected closed topological four-manifolds.