A PLATEAU PROBLEM FOR COMPLETE SURFACES IN THE DE-SITTER THREE-SPACE

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Abstract. In this paper we establish some existence and uniqueness theorems for a Plateau problem at infinity for complete spacelike surfaces in \( S^3_1 \) whose mean and Gauss–Kronecker curvatures verify the linear relationship
\[
2\varepsilon(H - 1) - (\varepsilon + 1)(K - 1) = 0 \quad \text{for} \quad -\varepsilon \in \mathbb{R}^+.
\]

1. Introduction

The global approach to surfaces with a constant curvature is a subject of many studies in Submanifolds Geometry, especially of that ones whose structure equations are integrable in terms of holomorphic data, because it represent a powerful tool in the study of these surfaces. Some representative examples are the Enneper–Weierstrass representation for minimal surfaces in \( \mathbb{R}^3 \) [13] and the McNertney–Kobayashi one for maximal surfaces in \( \mathbb{L}^3 \) presented in [9].

In this paper we will deal with spacelike surfaces in \( S^3_1 \), a topic developed in the recent years. For instance, in the compact case Ramanathan [14] proved that every compact spacelike surface in \( S^3_1 \) with constant mean curvature is totally umbilical. On the other hand, Li [10] showed that every compact spacelike surface in \( S^3_1 \) with constant Gaussian curvature is totally umbilical.

As a natural generalization of Ramanathan and Li results, Aledo and Gálvez [2] characterized the totally umbilical round spheres of \( S^3_1 \) as the only compact linear Weingarten spacelike surfaces.

In this work we study a special case of linear Weingarten surfaces of Bianchi type, in short BLW-surfaces, studied in [3]. We center our attention on BLW-surfaces whose mean and Gauss–Kronecker curvatures verify the linear relationship
\[
2\varepsilon(H - 1) - (1 + \varepsilon)(K - 1) = 0, \quad -\varepsilon \in \mathbb{R}^+.
\]