THE BIHARMONIC STRESS-ENERGY TENSOR
AND THE GAUSS MAP

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Abstract. We consider the energy and bienergy functionals as variational
problems on the set of Riemannian metrics and present a study of the bi-
harmonic stress-energy tensor. This approach is then applied to characterize
weak conformality of the Gauss map of a submanifold. Finally, working
at the level of functionals, we recover a result of Weiner linking Willmore
surfaces and pseudo-umbilicity.

1. Introduction

The guiding principle of variational theory is that geometric objects can be selected
according to whether or not they minimize certain functionals and, since Morse
theory, critical points can prove sufficiency. Once this criterion is chosen, the ade-
quate Euler–Lagrange equation will characterise maps particularly well adapted
to our geometric framework. However, roles can be reversed and metrics can be
viewed as variables and required to fit with a map and complete the picture. Other
than the duality of these approaches, the theory of general relativity has put met-
rics firmly in centre of the stage and the characterisation of Einstein metrics as
(constrained) critical points of the total curvature has created a new viewpoint on
the usual functionals, in particular the various energies defined for maps between
manifolds.

Let \( \phi : (M, g) \rightarrow (N, h) \) be a smooth map between Riemannian manifolds
of dimension \( m \), respectively \( n \). Assuming that \( M \) is compact we can define the