ON THE TRANSLATIONALLY-INVARIANT SOLUTIONS OF THE MEMBRANE SHAPE EQUATION

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Abstract. The membrane shape equation derived by Helfrich and Ou-Yang describes the equilibrium shapes of biomembranes, built by bilayers of amphiphilic molecules, in terms of the mean and Gaussian curvatures of their middle-surfaces. Here, we present a new class of translationally-invariant solutions to this equation in terms of the elliptic functions which completes the solutions found earlier. In this way, all translationally-invariant solutions to the membrane shape equation are determined. Special attention is paid to those translationally-invariant solutions of the membrane shape equation which determine closed cylindrical (tube-like) surfaces (membrane shapes). Several examples of such surfaces are presented.

1. Introduction

Within the framework of the Helfrich spontaneous curvature model [3], the equilibrium shapes of a biomembrane, assumed as a bilayer of amphiphilic molecules (phospholipids, for instance), are described in terms of the mean $H$ and Gaussian $K$ curvatures of its middle-surface $S$ by the membrane shape equation [7, 8]

$$2k_c \Delta H + k_c (2H + \mathbb{H}) (2H^2 - \mathbb{H}H - 2K) - 2\lambda H + p = 0$$

(1)

where $k_c$, $\mathbb{H}$ and $\lambda$ are real constants representing the bending rigidity, spontaneous curvature and tensile stress of the membrane, respectively, while $p$ is the osmotic pressure difference between the outer and inner media assumed to be a real constant too. Here, $\Delta$ is the Laplace–Beltrami operator on the surface $S$. 

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